Event-based control using quantized state information

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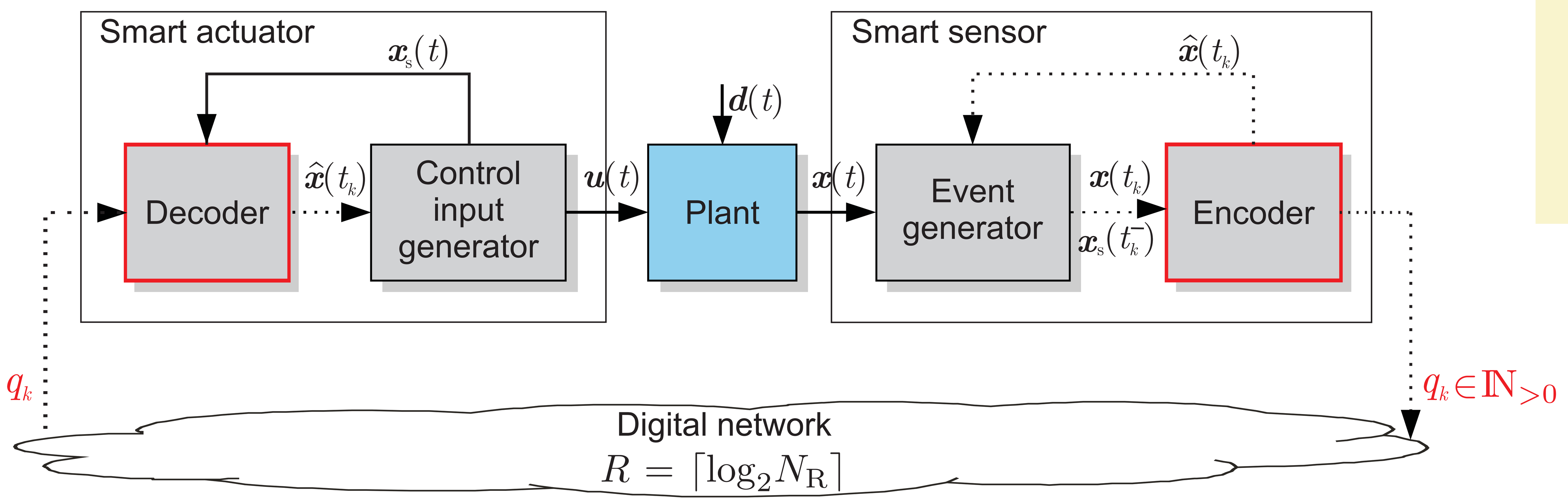
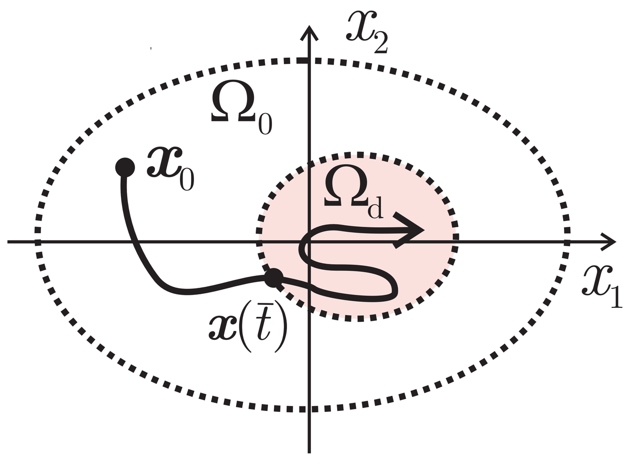
Structure of the event-based control loop



Goals:
- Reduction of information exchange
- Reduction of information content
- Ultimate boundedness



Basic components

Plant

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{E}\boldsymbol{d}(t), \quad \boldsymbol{x}(0) = \boldsymbol{x}_0$$

Control input generator

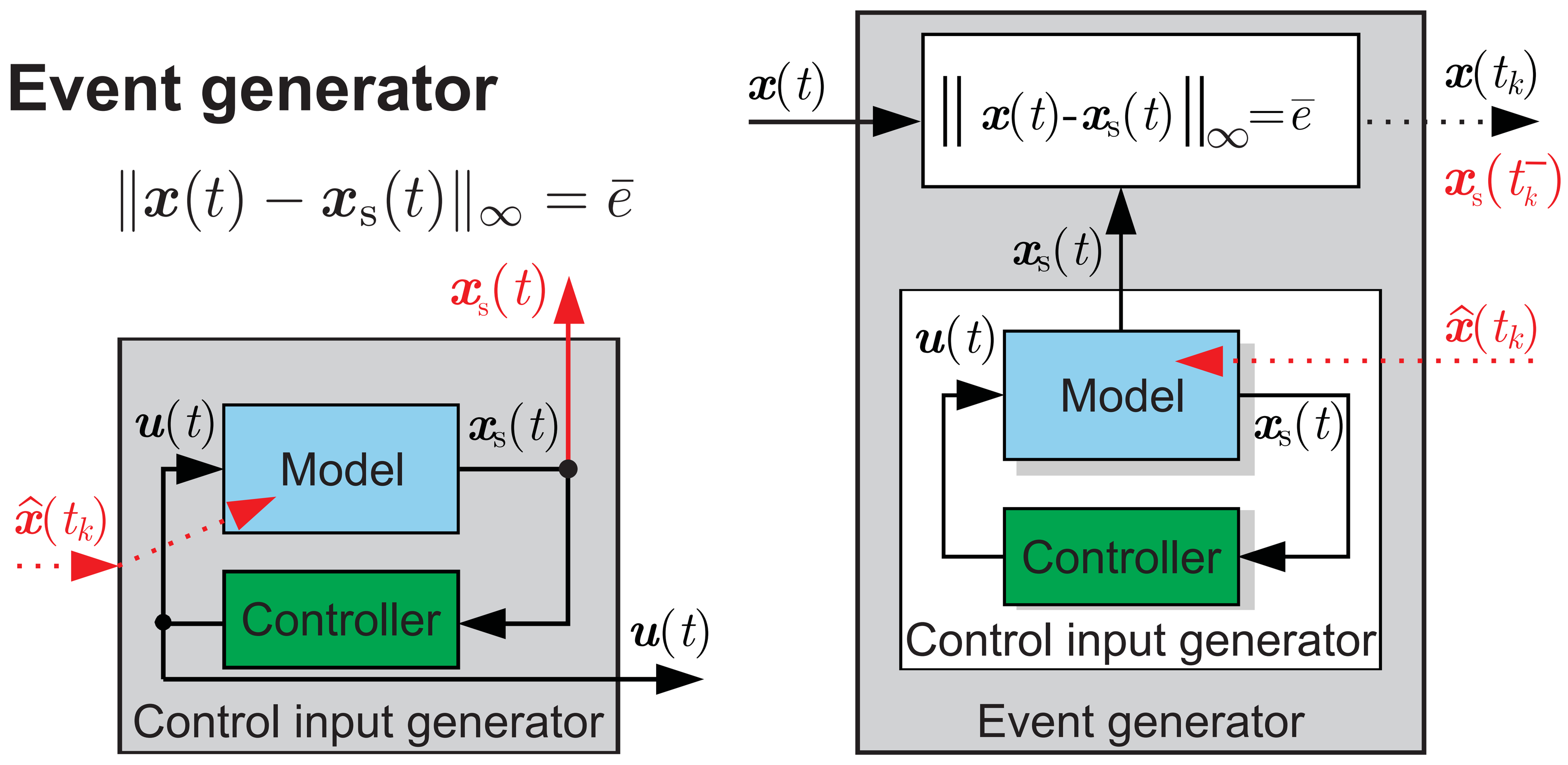
$$\dot{\boldsymbol{x}}_s(t) = (\boldsymbol{A} - \boldsymbol{B}\boldsymbol{K})\boldsymbol{x}_s(t), \quad t_k \le t < t_{k+1}$$

$$\boldsymbol{x}_s(t_k^+) = \hat{\boldsymbol{x}}(t_k)$$

$$\boldsymbol{u}(t) = -\boldsymbol{K}\boldsymbol{x}_s(t)$$

Event generator

$$\|\boldsymbol{x}(t) - \boldsymbol{x}_s(t)\|_\infty = \bar{e}$$



Quantization mechanism

Encoder

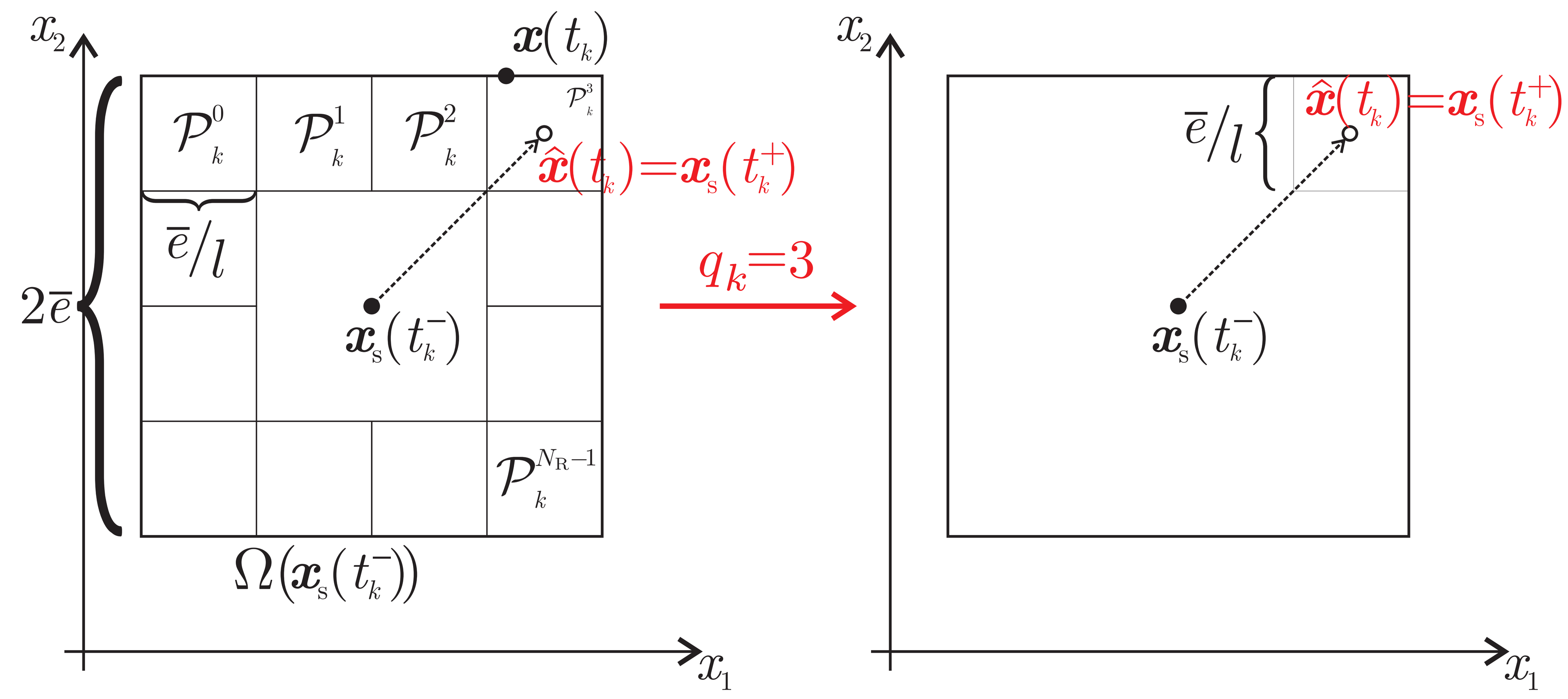
1. Subdivision of $\Omega(\boldsymbol{x}_s(t_k^-)) \subset \mathbb{R}^n$
   $\Rightarrow q \in \{0, 1, \ldots, N_R - 1\}$,
   $N_R = l^n - (l-2)^n$
2. Determination of $\mathcal{P}_k^q \ni \boldsymbol{x}(t_k)$
3. Transmission of q_k

Decoder

1. Reconstruction of the center point $\hat{\boldsymbol{x}}(t_k) \in \mathcal{P}_k^q$

$$\Rightarrow \|\boldsymbol{x}(t_k) - \hat{\boldsymbol{x}}(t_k)\|_\infty = \frac{\bar{e}}{l}$$

2. Provision of the state $\hat{\boldsymbol{x}}(t_k)$
   $\boldsymbol{x}_s(t_k^+) = \hat{\boldsymbol{x}}(t_k)$



Main results

The difference $e(t) = \boldsymbol{x}(t) - \boldsymbol{x}_{SF}(t)$ between the states of the event-based control loop and the continuous-time state-feedback loop is bounded by:

$$\|\boldsymbol{x}(t) - \boldsymbol{x}_{SF}(t)\|_\infty \le e_{\max} = \bar{e} \cdot \int_0^\infty \| \mathrm{e}^{\bar{\boldsymbol{A}}\tau} \boldsymbol{B}\boldsymbol{K} \|_\infty \, \mathrm{d}\tau$$

For any bounded disturbance, the minimum time $T_{\min}$ between two communication events is bounded from below by $\bar{T}_d$ given by:

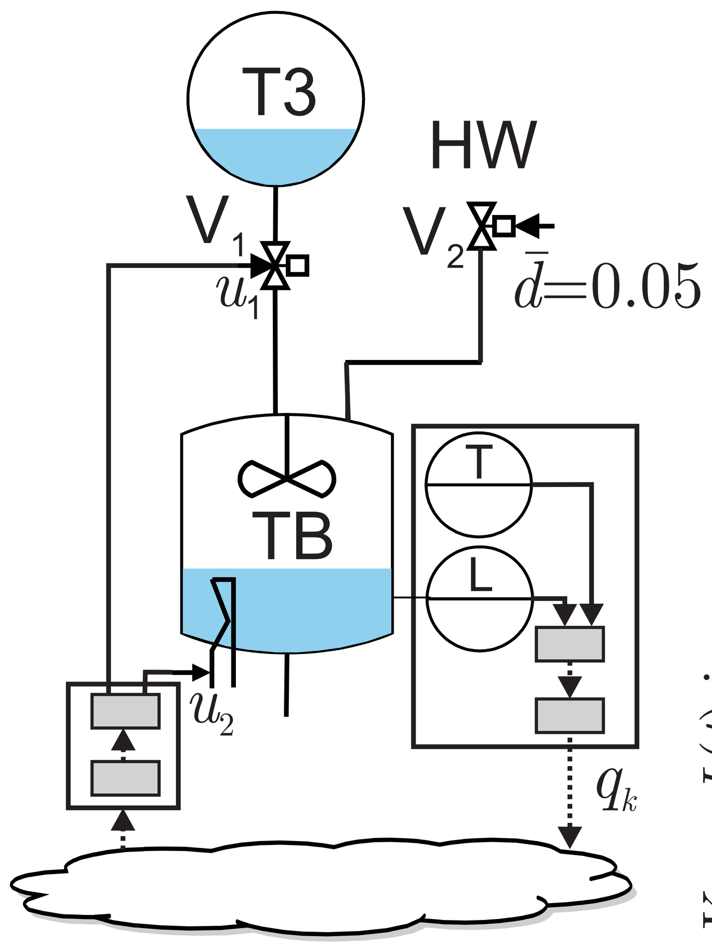
$$\int_0^{\bar{T}_d} \left\| \mathrm{e}^{\boldsymbol{A}t} \boldsymbol{E} \right\|_\infty \mathrm{d}\tau = \frac{\bar{e}}{d_{\max}} \cdot \left(1 - \frac{\max_t \left\| \mathrm{e}^{\boldsymbol{A}t} \right\|_\infty}{l}\right)$$

In the undisturbed case, the minimum time $T_{\min}$ between two communication events is bounded from below by $\bar{T}$ given by:

$$\left\| \mathrm{e}^{\boldsymbol{A}\bar{T}} \right\|_\infty = l$$

Application

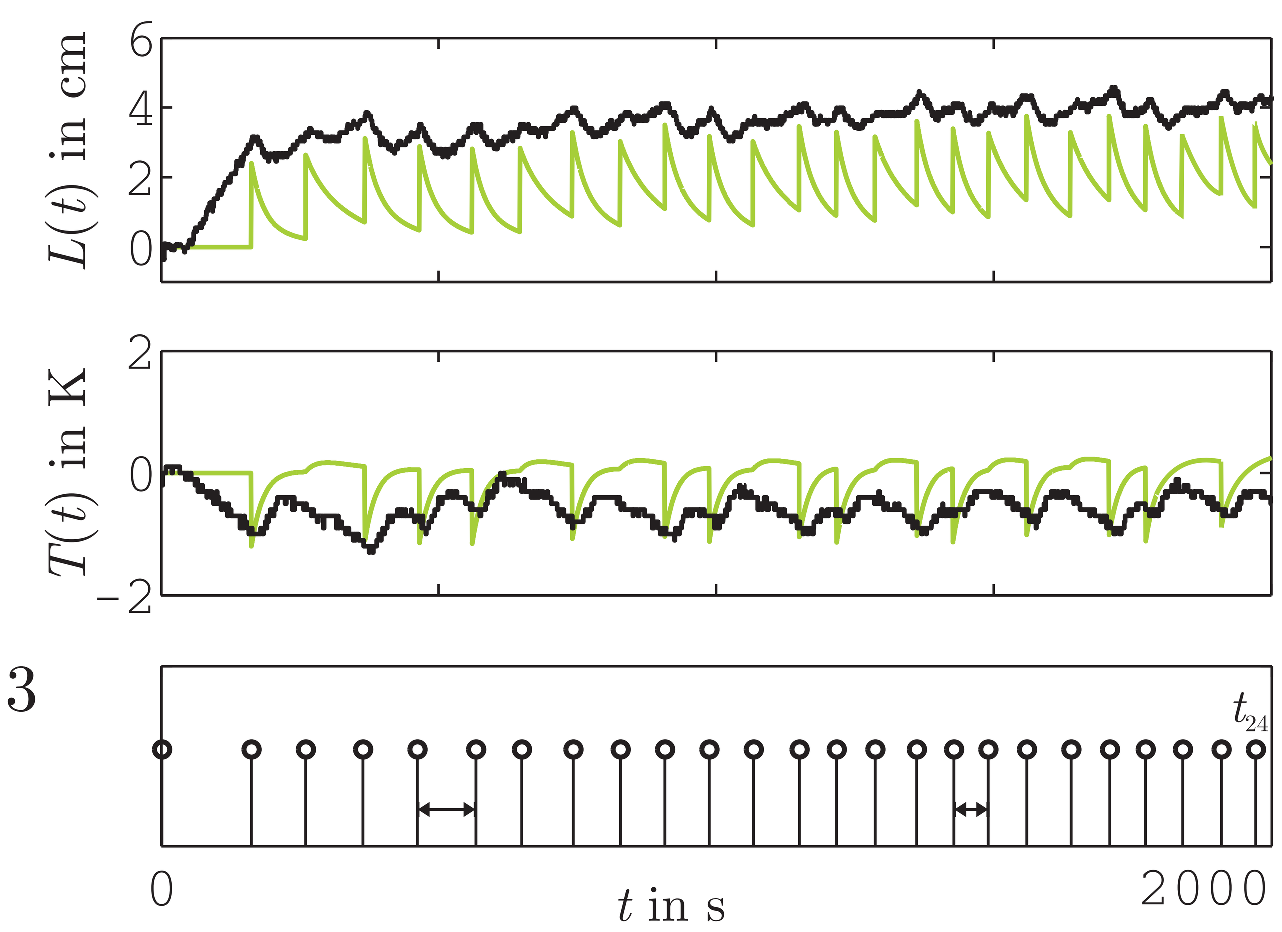
Plant



Model

$$\dot{\boldsymbol{x}}(t) = \begin{pmatrix} -0.8 \times 10^{-3} & 0 \\ 1 \times 10^{-10} & -1.8 \times 10^{-3} \end{pmatrix} \boldsymbol{x}(t) + \begin{pmatrix} 0.211 & 0 \\ -0.112 & 0.023 \end{pmatrix} \boldsymbol{u}(t) + \begin{pmatrix} 0.148 \\ -0.081 \end{pmatrix} d(t)$$

Experimental results



Event generator

$$\|\boldsymbol{x}(t) - \boldsymbol{x}_s(t)\|_\infty = 3$$

Encoder

$$l = 5 \;\Rightarrow\; N_R = 16$$