A DMPC applied to multisource temperature regulation in buildings

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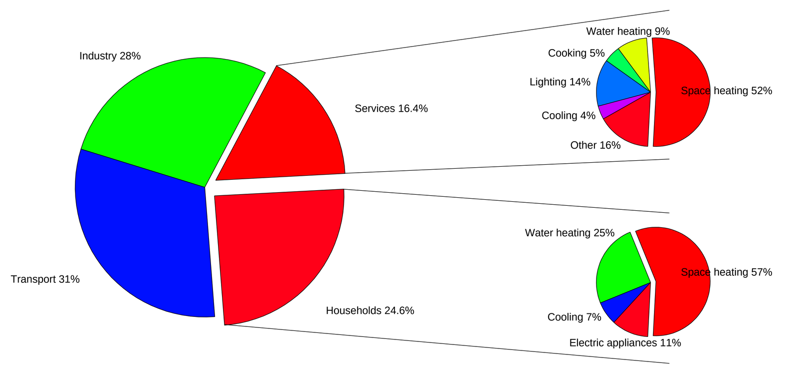
1. Introduction. Economical context

- World objectives: reduce the running costs / energy consumption without affecting the quality of life
- Building heating systems: ≈22.5% of total energy consumption in EU

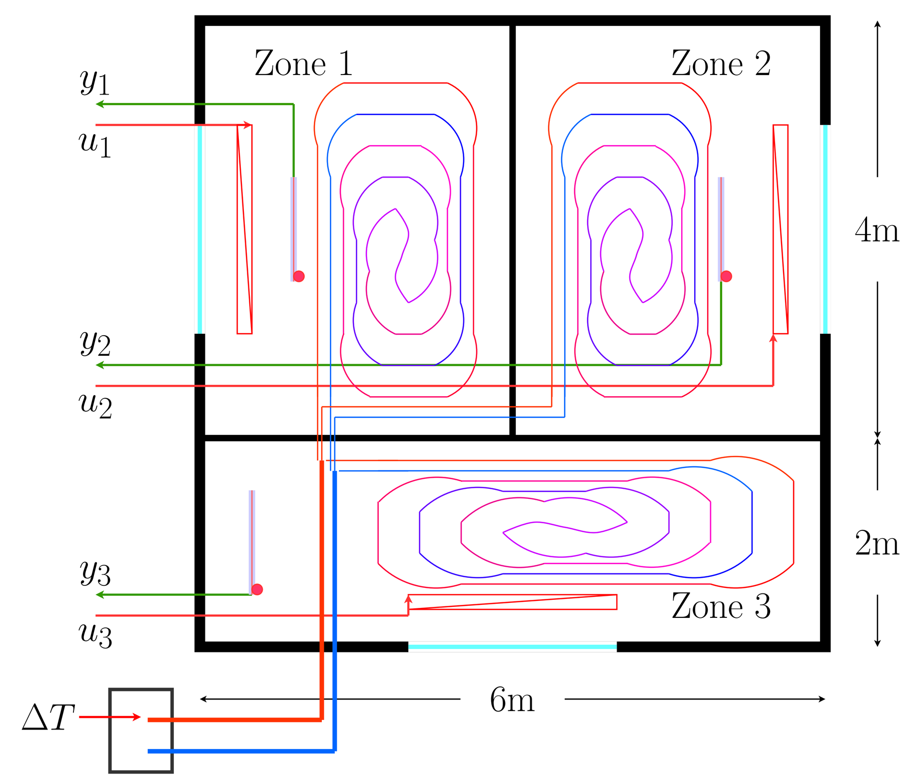


- Solutions:
  - Construct new low energy buildings (<1%)
  - Optimize the control of HS in existing buildings
- Control problem: minimize the heating cost while maintaining a certain thermal comfort during the occupation periods
- MPC choice motivation:
  - Optimality relative to a criterion
  - Constraints explicitly considered
  - Anticipative effect, exploiting the knowledge of future temperature / occupation profile

2. System description

- Zone i model:

$$\begin{cases} \boldsymbol{x}_i(k+1) = \boldsymbol{A}_i\boldsymbol{x}_i(k) + \boldsymbol{B}_{ic}u_c(k) + \sum_{j\in\hbar_i\cup\{i\}} \boldsymbol{B}_{ij}u_j(k) \\ y_i(k) = \boldsymbol{C}_i\boldsymbol{x}_i(k) \end{cases}$$



- Considering the thermal coupling between adjacent zones
- Heating sources:
  - Local electric sources: fast dynamics & high price
  - Central gas source: slow dynamics & low price

3. MPC formulation

- Global optimization problem:

$$\min_{\boldsymbol{u}_c(k),\boldsymbol{u}_i(k),\forall i\in S} J(k) = \sum_{i\in S}\left(\boldsymbol{c}_i^T(k)\boldsymbol{u}_i(k) + \sum_{j=1}^{N_2} f_i(k+j)\right) + \boldsymbol{c}_c^T(k)\boldsymbol{u}_c(k),$$

subject to (hard control input constraints)

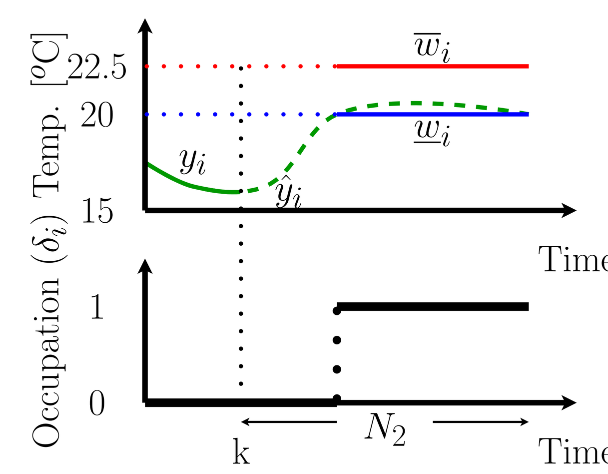
$$0 \le u_i(k+j) \le \overline{u}_i,\ \forall j = 0...N_u-1,\ \forall i \in S,$$

$$0 \le u_c(k+j) \le \overline{u}_c,\ \forall j = 0...N_u-1,$$

with (soft output constraints)

$$f_i(k+j) = \begin{cases} 0,\ \mu_i^{up}(k+j) \le 0 \text{ and } \mu_i^{low}(k+j) \le 0 \\ \lambda_i\mu_i^{up}(k+j),\ \mu_i^{up}(k+j) > 0 \\ \lambda_i\mu_i^{low}(k+j),\ \mu_i^{low}(k+j) > 0, \end{cases}$$

$$\mu_i^{low}(k+j) = \delta_i(k+j)(\underline{w}_i(k+j) - \hat{y}_i(k+j|k)),$$
$$\mu_i^{up}(k+j) = \delta_i(k+j)(\hat{y}_i(k+j|k) - \overline{w}_i(k+j)),$$
$$\forall i \in S,\ \forall j = 1...N_2$$



4. DMPC via Benders' decomposition

- Equivalent LP problem in standard form:

$$\begin{array}{lllllllll} \min\limits_{\boldsymbol{u}'_c,\boldsymbol{u}'_i,\forall i\in S} & \boldsymbol{c}_c'^T\boldsymbol{u}'_c & + & \boldsymbol{c}_1'^T\boldsymbol{u}'_1 & \cdots & + & \boldsymbol{c}_s'^T\boldsymbol{u}'_s & & \\ \text{subject to} & \boldsymbol{D}\boldsymbol{u}'_c & & & & & & = & \boldsymbol{g} \\ & \boldsymbol{E}_1\boldsymbol{u}'_c & + & \boldsymbol{F}_{11}\boldsymbol{u}'_1 & \cdots & + & \boldsymbol{F}_{1s}\boldsymbol{u}'_s & = & \boldsymbol{h}_1 \\ & \vdots & & & & \ddots & & & \vdots \\ & \boldsymbol{E}_s\boldsymbol{u}'_c & + & \boldsymbol{F}_{s1}\boldsymbol{u}'_1 & \cdots & + & \boldsymbol{F}_{ss}\boldsymbol{u}'_s & = & \boldsymbol{h}_s \\ & \boldsymbol{u}'_c & , & \boldsymbol{u}'_1 & \cdots & , & \boldsymbol{u}'_s & \ge & \boldsymbol{0}, \end{array}$$

- Master problem (MP):

$$\min_{\boldsymbol{u}'_c,z} \boldsymbol{c}_c'^T\boldsymbol{u}'_c + z,$$

subject to

$$\boldsymbol{D}\boldsymbol{u}'_c = \boldsymbol{g},\ \boldsymbol{u}'_c \ge \boldsymbol{0},\ z \ge 0,$$

$$\sum_{i=1}^{s}(\overline{\boldsymbol{p}}_i^{l_p})^T\boldsymbol{E}_i\boldsymbol{u}'_c + z \ge \sum_{i=1}^{s}(\overline{\boldsymbol{p}}_i^{l_p})^T\boldsymbol{h}_i,\ \forall l_p = 1...l-1.$$

- Primal subproblem (PSP):

$$z_i(\boldsymbol{u}'_c) = \min_{\boldsymbol{u}'_i} \boldsymbol{c}_i'^T\boldsymbol{u}'_i,$$

subject to

$$\boldsymbol{F}_{ii}\boldsymbol{u}'_i = \boldsymbol{h}_i - \boldsymbol{E}_i\boldsymbol{u}'_c - \sum_{j\in\hbar_i}\boldsymbol{F}_{ij}\boldsymbol{u}_j'^{l_i-1},$$

$$\boldsymbol{u}'_i \ge \boldsymbol{0}.$$

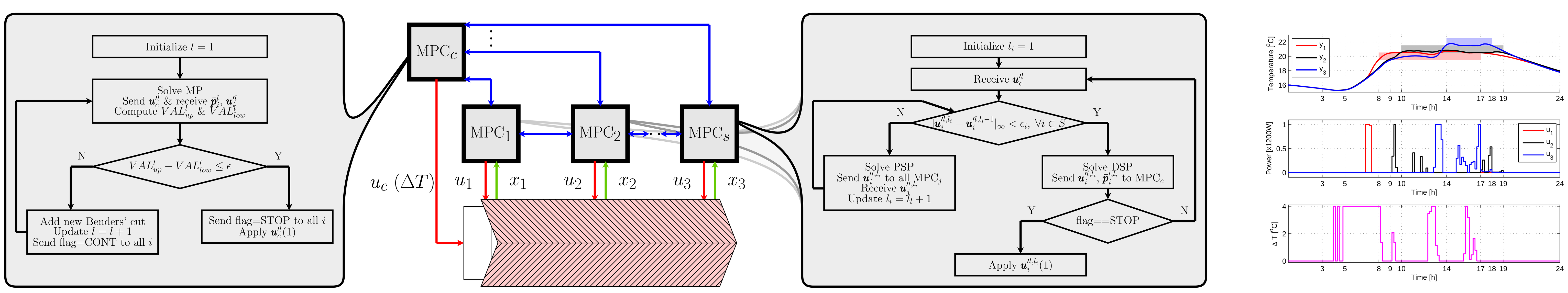
- Dual subproblem (DSP):

$$z_i(\boldsymbol{u}'_c) = \max_{\boldsymbol{p}_i} \boldsymbol{p}_i^T(\boldsymbol{h}_i - \boldsymbol{E}_i\boldsymbol{u}'_c - \sum_{j\in\hbar_i}\boldsymbol{F}_{ij}\boldsymbol{u}_j'^{l_i-1}),$$

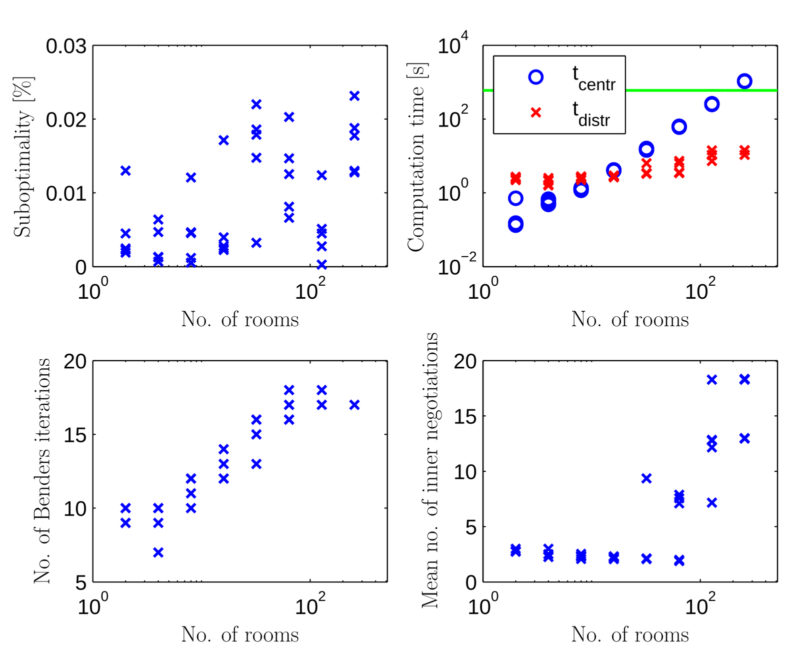
subject to

$$\boldsymbol{F}_{ii}^T\boldsymbol{p}_i \le \boldsymbol{c}'_i.$$

5. Distributed control scheme & algorithm. Qualitative results



6. Statistical results



7. Quantitative results

- Simulation parameters:
  - Weather conditions: Rennes, January 1^{st}, 1998
  - Convector's maximal power: 1200W
  - Boiler efficiency: $\eta = 0.9$ (ideal boiler behavior)
  - Electricity price: $c_e = 0.0742$ €/kWh
  - Gas price: $c_g = 0.063$ €/kWh

Control law	Cost [€]
Coupled model - centralized MPC	74.69
Coupled model - DMPC	74.89
Decoupled model - DMPC	78
Local PIs and open loop control for ΔT	100

8. Conclusions

- MPC takes advantage of knowing in advance the occupation profile & temperature comfort bounds
- Control law: online LP optimization
- Important money save (25% comparing to current practice)
- Solution for large-scale systems (buildings):
  - Scalable distributed control scheme
  - Two-level iteration algorithm
  - Reduction of computational demand by parallelization