A Model Predictive Control Approach for Stochastic Networked Control Systems

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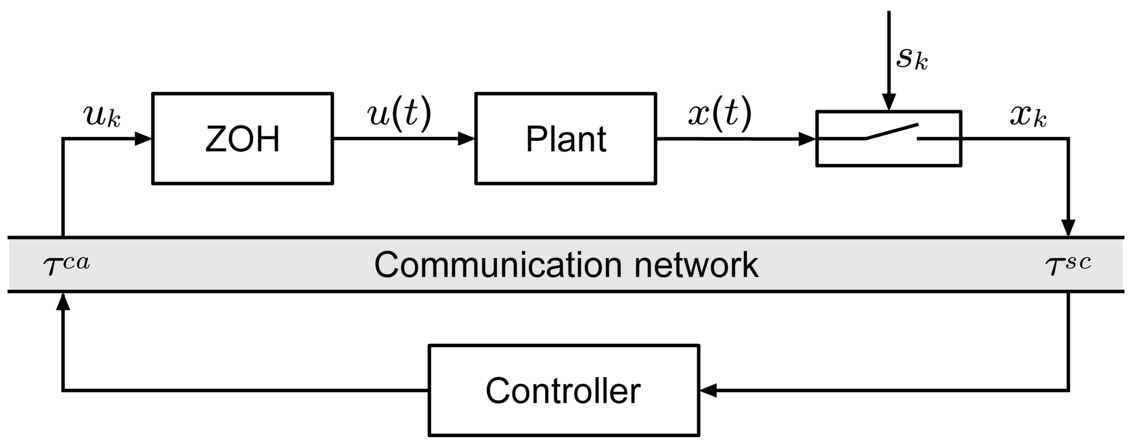
Contribution

We introduce a **stochastic model predictive control (SMPC)** approach for networked control systems (NCSs) that are subject to time-varying sampling intervals and time-varying transmission delays. Assuming that the controlled plant can be modeled as a linear system, we present a SMPC formulation based on scenario enumeration that optimizes a stochastic performance index and provides closed-loop stability in the mean-square sense.

NCS Model



The considered NCS model includes:

- A continuous-time linear **plant** of the form

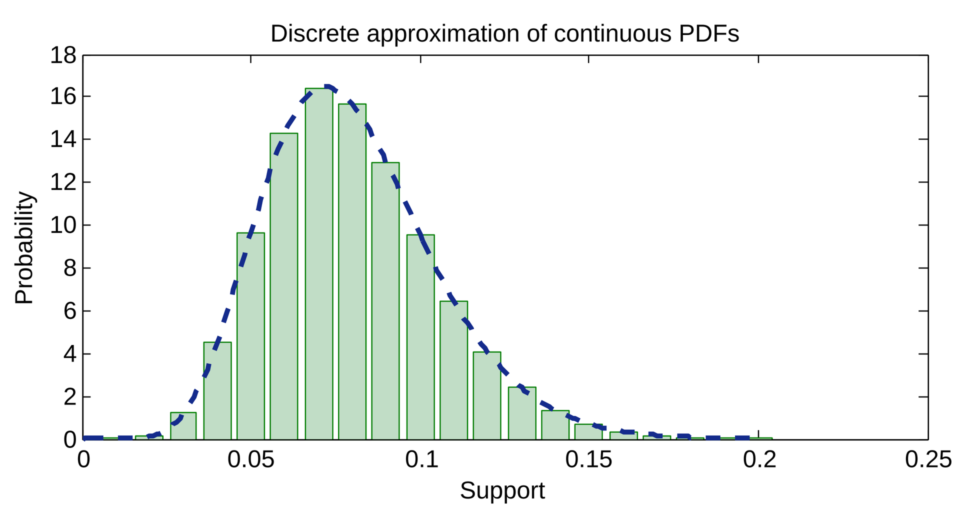
$$\dot{x}(t) = Ax(t) + Bu(t) \qquad (1)$$

- Time-varying **sampling intervals** h_k
- Time-varying **delays** τ_k

The uncertain parameters (h_k, τ_k) are assumed bounded, with $\tau_k \le h_k$, and described by an arbitrary **continuous probability distribution**. By discretizing the plant (1) at the sampling times s_k and using $\xi_k = [x_k^T \; u_{k-1}^T]^T$, the NCS is formulated as

$$\xi_{k+1} = \tilde{A}_{h_k,\tau_k}\xi_k + \tilde{B}_{h_k,\tau_k}u_k \qquad (2)$$

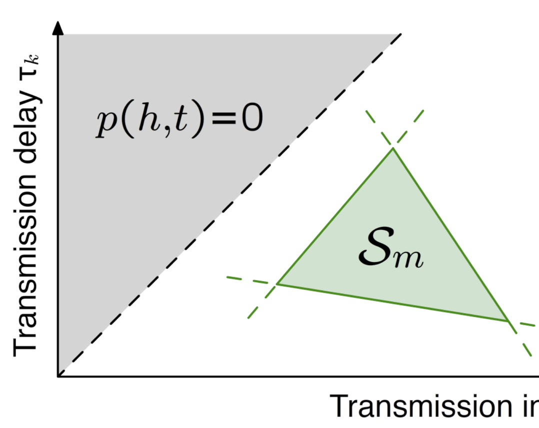
Overapproximation of NCS



- Following the approach of [1], the uncertain parameters set is **partitioned** in a number of regions, and for every region a local **overapproximation** of dynamics (2) is computed.
- The problem of finding a function $V(\xi_k) = \xi_k^T P \xi_k$ which grants mean-square stability can now be recast as

$$\mathbb{E}[V(\xi_{k+1})] \le \sum_{m=1}^{S} p_m \max_{(h_k,\tau_k)\in\mathcal{S}_m} \xi_k^T C_{h_k,\tau_k}^T P C_{h_k,\tau_k}\xi_k$$

- This can be converted into **LMI conditions**.



$$\Pr\left((h,\tau)\in\mathcal{S}_m\right) = \iint_{\mathcal{S}_m} p(h,\tau)dhd\tau$$

Stochastic MPC design

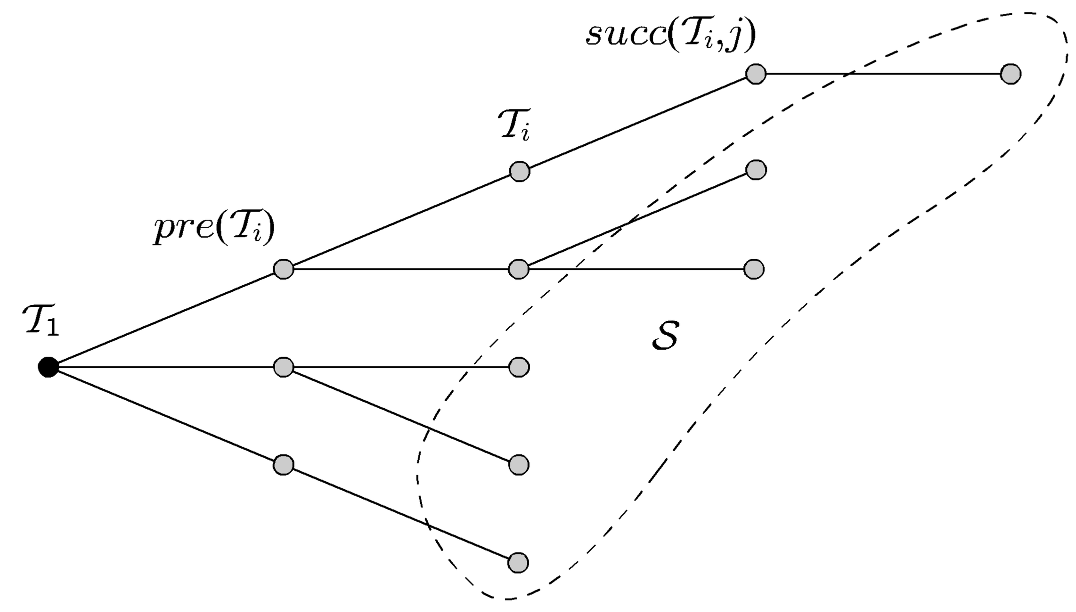
The SMPC policy is derived from the one presented in [2], and relies on the following steps:

- First, a new **partition** $\phi_1, \phi_2, \dots, \phi_s$ of the parameters space is defined, and a realization probability is associated to every region.
- The **prediction model** for the MPC controller is given by the collection of the averaged dynamics of the NCS (2) in every region ϕ_i, i.e.,

$$\xi_{k+1} = \begin{cases} \bar{A}_1\xi_k + \bar{B}_1 u_k & \text{if } (h_k,\tau_k)\in\phi_1, \\ \bar{A}_2\xi_k + \bar{B}_2 u_k & \text{if } (h_k,\tau_k)\in\phi_2, \\ \quad\vdots & \quad\vdots \\ \bar{A}_s\xi_k + \bar{B}_s u_k & \text{if } (h_k,\tau_k)\in\phi_s, \end{cases}$$

$$\bar{A}_n = \iint_{\phi_n}\tilde{A}_{h,\tau}p(h,\tau)dhd\tau,\; \bar{B}_n = \iint_{\phi_n}\tilde{B}_{h,\tau}p(h,\tau)dhd\tau.$$

This allows a **decoupling** between stability requirements and performance optimization.
- Then, an **optimization tree** based on the prediction model is designed, following a maximum likelihood policy. Every node is identified by a predicted state and input. This leads to a **multiple-horizon** control problem.



- Offline, a **Lyapunov function** which provides mean-square stability is obtained by exploiting the NCS overapproximation and imposing a state-feedback structure on the input.
- Online, a quadratically constrained quadratic problem (QCQP) based on the precomputed optimization tree is solved. The objective function is an approximation of the **closed-loop expected trajectory**. The problem incorporates quadratic constraints to enforce **mean-square stability**.

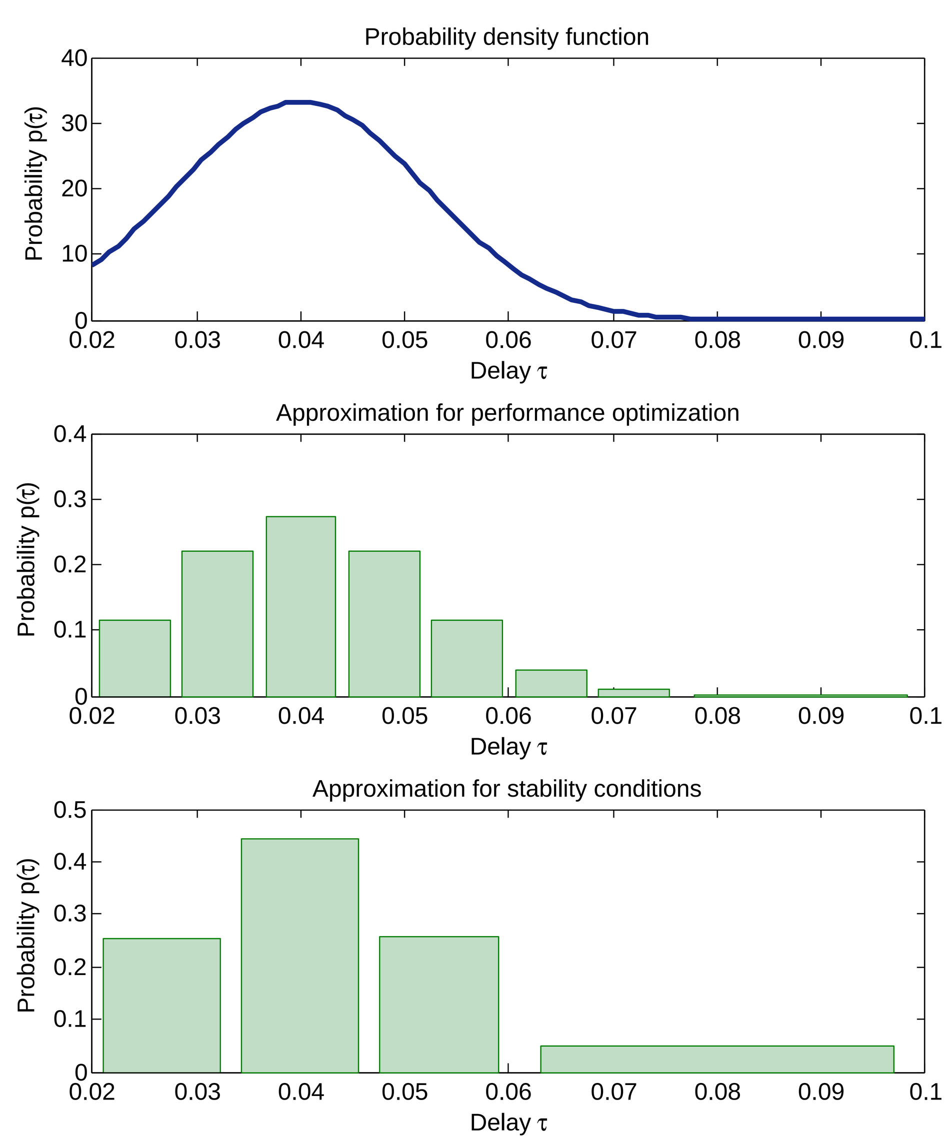
Illustrative example

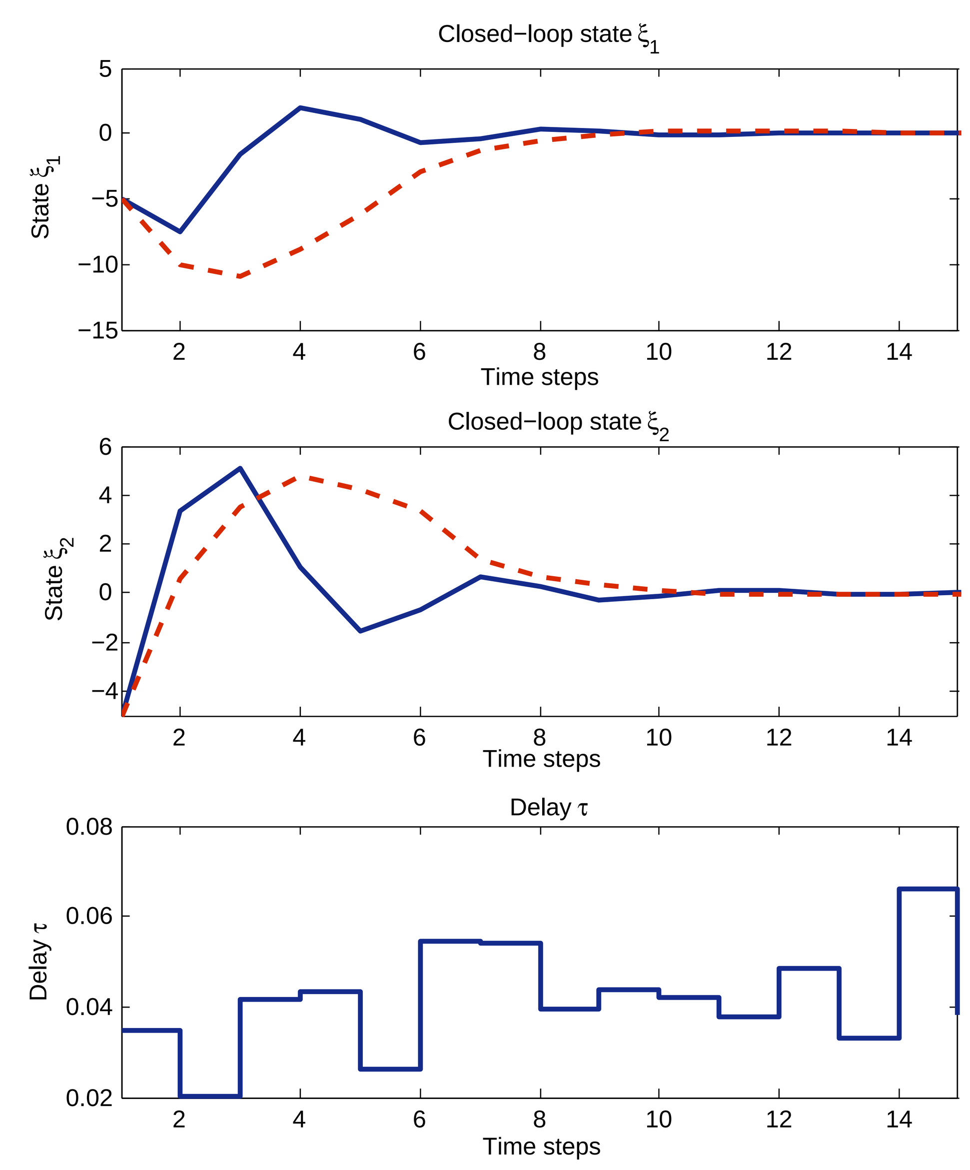
The SMPC scheme was tested on an open-loop unstable plant, modeled as (1) with $A = \left[\begin{smallmatrix} 1 & 15 \\ -15 & 1 \end{smallmatrix}\right]$ and $B = \left[\begin{smallmatrix} 0.2 \\ 0.8 \end{smallmatrix}\right]$. The sampling intervals h_k are taken constant and equal to $h_{\text{nom}} = 0.1$, while the PDF modeling the realizations of the delay τ_k is given by a truncated normal distribution.

We construct $S = 4$ line segments to partition the set of possible values of τ_k. This allows us to obtain a (mean-square) stabilizing controller of the form $u_k = K\xi_k$. With the aim of improving performances, we perform a finer partition for prediction purposes, using $s = 8$ line segments.

We run 100 simulations comparing the SMPC with a **robust state-feedback** controller (RSF), that provides robust convergence to the origin.

Controller	$\mu(J_i)$	$\sigma(J_i)$	avg. time
RSF	884.34	382.19	–
SMPC	678.01	134.74	29 ms





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