Continuous-time double integrator consensus algorithms improved by an appropriate sampling

gipsa-lab

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FeedNeback co-design for networked control systems

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Introduction

A "consensus" algorithm (or protocol) is an interaction rule that specifies the information exchange between an agent and all of its neighbors over the network in order to reach an agreement regarding a certain quantity of interest that depends on the state of all agents.

.Agents are assumed to obey a double integrator model
.A novel consensus algorithm based on a sampling approach is provided
.A method to design the algorithm parameters, including the appropriated sampling period T, on an "optimal" way is proposed based on a LMI's formulation.

Problem Statement

Consider the classical double integrator consensus algorithm :

$$\ddot{x}(t) = -\sigma\dot{x}(t) - Lx(t)$$

where x represents the vector containing the agents variables. If $\sigma = 0$,

$$\ddot{x}(t) = -Lx(t)$$

By introducing the augmented vector $y(t) = [x^T(t)\ \dot{x}^T(t)]^T$, we have

$$\dot{y}(t) = \begin{bmatrix} 0 & I \\ -L & 0 \end{bmatrix} y(t) = \bar{L}y(t)$$

Main Idea: In some cases delays can help to stabilize systems which are unstable without them

Proposed algorithm

Systems with

Constant Delay	"Memory" effect
Sampled Delay	No "Memory" effect

The previous algorithm is modified into a new algorithm defined by :

$$\ddot{x}(t) = -(L + \delta^2 I)x(t) + \delta^2 x(t_k)$$

with $\tau(t) = t - t_k,\ t_k \le t < t_{k+1}$. If we take x= Wz, then $ULW = \begin{bmatrix} \Delta & \vec{0} \\ \vec{0}^T & 0 \end{bmatrix}$.

The previous consensus problem can be rewritten as:

$$\ddot{z}_1(t) = -(\Delta + \delta^2 I)z_1(t) + \delta^2 z_1(t_k)$$
$$\ddot{z}_2(t) = -\delta^2 z_2(t) + \delta^2 z_2(t_k)$$

Then the dynamics of z_1 can be rewritten as follows

$$\dot{y}(t) = A(\delta)y(t) + A_d(\delta)y(t_k)$$

where $A(\delta) = \begin{bmatrix} 0 & I \\ -(\Delta + \delta^2 I) & 0 \end{bmatrix}$, $A_d(\delta) = \begin{bmatrix} 0 & 0 \\ \delta^2 I & 0 \end{bmatrix}$

Main Result: Stability Analysis

Consider the proposed consensus algorithm. Assume that there exist $P > 0$, $R > 0$, S_1, S_2, N and X that satisfy:

$$\Pi_1 + h_\alpha(T,0)M_2^T X M_2 + f_\alpha(T,0)\Pi_2 < 0$$

$$\begin{bmatrix} \Pi_1 + h_\alpha(T,T)M_2^T X M_2 & g_\alpha(T,T)N \\ * & -g_\alpha(T,T)R \end{bmatrix} < 0$$

where $M_0 = [\ A(\delta)\ \ A_d(\delta)\ ]$, $M_1 = [\ I\ \ 0\ ]$, $M_2 = [\ 0\ \ I\ ]$, $M_3 = [\ I\ \ -I\ ]$ and

$$\Pi_1 = 2M_1^T P(M_0 + \alpha M_1) - M_3^T(S_1 M_3 + 2S_2 M_2) - 2NM_3$$
$$\Pi_2 = M_0^T(RM_0 + 2S_1 M_3 + 2S_2 M_2)$$

The functions f_α, g_α and h_α for all scalars T and $\tau \in [0\ T]$ are given by

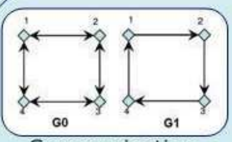
$$f_\alpha(T,\tau) = (e^{2\alpha(T-\tau)} - 1)/2\alpha,$$
$$g_\alpha(T,\tau) = e^{2\alpha T}(1 - e^{-2\alpha\tau})/2\alpha,$$
$$h_\alpha(T,\tau) = -1 - 2\alpha f_\alpha(T,\tau) - f_\alpha(T,0)/T$$

Then, the consensus algorithm is thus $a_g - stable$, where $\alpha_g = \min\{\alpha, -\log(\cos(\delta T))\}$.
Moreover, the consensus equilibrium is given by

$$x(\infty) = U_2\,(x(0) + \gamma_{\delta T}\dot{x}(0)),$$

with $\gamma_{\delta T} = \sin(\delta T)/(\delta(1 - \cos(\delta T))) = \tan((\pi - \delta T)/2)/\delta$

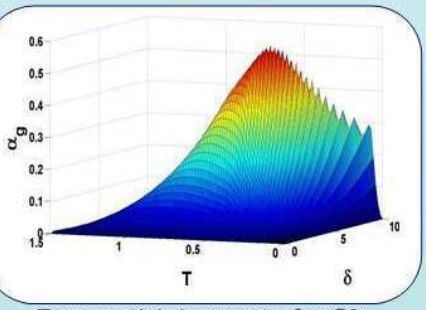
Performances Simulations and Conclusions



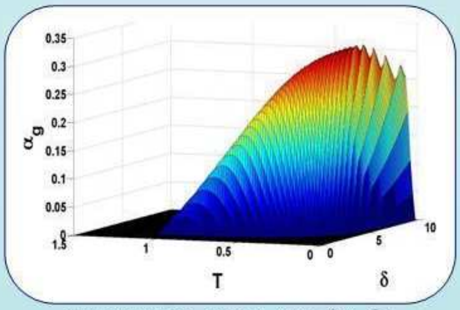
Communication graphs Topology

In the proposed algorithm

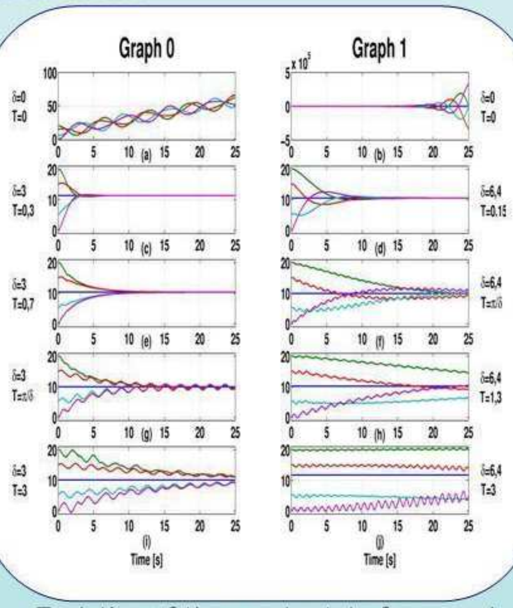
.Reduction of information quantity needed for control
.No more need of velocity sensors
.Economical, space and calculation savings
.Exponential stability of the solutions is achieved



Exponential decay rate for G0



Exponential decay rate for G1



Evolution of the agents state for several values of the sampling period T

Drawbacks

The proposed stability criteria expressed in term of LMIs complexity will drastically increase for large networks.