Smooth Transitions between Trajectory Tracking and Path Following for Single Vehicles and Formations

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Abstract

Trajectory tracking and path following both have their benefits, and depending on the mission of a single vehicle or a formation, one may be more favourable than the other. Path manoeuvring can be seen as a combination of both, and has the potential to combine their benefits. We present a method that makes it possible to smoothly change between trajectory tracking, path manoeuvring, and path following; this can even be done during an active mission. The method is presented for both single vehicles and formations. The theoretically obtained results are supported by simulations.

Introduction

In order to follow a trajectory—by which we mean a parametrised geometric curve $\mathbf{c}(s)$ describing the desired positions (and orientations) to follow—we can identify two separate tasks:

- *Geometric task:* Force the position (and orientation) $\eta(t)$ to converge to the path $\mathbf{c}(s(t))$.
- *Dynamic task:* Follow the desired path $\mathbf{c}(s(t))$ at its desired along-path speed, possibly dictated by strict time-dependence of the desired position along the path.

In literature mostly a choice is made between using path following or trajectory tracking, while recently path manoeuvring is discussed as well. We interpret these three terms as follows:

- *Path following:* Follow a geometric curve $\mathbf{c}(s)$ with no time-dependence.
- *Trajectory tracking:* Follow a geometric curve $\mathbf{c}(t)$ with a strict time-dependence.
- *Path manoeuvring:* Follow a geometric curve $\mathbf{c}(s(t))$ with a flexible time-dependence.

Using Virtual Times

We suggest the use of a path-manoeuvring controller, based on the design of a trajectory-tracking controller, and made more flexible by the substitution of a **virtual time** s for the real time t. We define the **time-difference**

$$\tilde{s} := t - s \quad \Rightarrow \quad \dot{\tilde{s}} = 1 - \dot{s}, \tag{1}$$

where the update law for the virtual time is

$$\dot{s} = f(d)\,[1 + \alpha\,(\beta\tilde{s} - 1)] + \alpha, \tag{2}$$

where $\alpha, \beta > 0$ are scalar tuning parameters discussed later, and $f(d)$ is a decreasing function in the distance $d := \sqrt{(x_\mathrm{t} - x)^2 + (y_\mathrm{t} - y)^2}$ between the vessel and its target, satisfying the properties

$$f(0) = 1, \quad \lim_{d\to\infty} f(d) = 0, \quad \frac{\partial}{\partial d} f(d) < 0. \tag{3}$$

Formation Synchronisation

The update law (2) can be adjusted to include synchronisation for formations of n vehicles. In order to do so, we define the **synchronisation error** for vehicle i as

$$\bar{s}_i = \frac{1}{n}\sum_{j=1}^{n} s_j - s_i, \tag{4}$$

where s_j are the virtual times of the n vehicles. Using a function $g(\bar{s}_i)$ satisfying

$$g(0) = 0, \qquad \bar{s}_i g(\bar{s}_i) > 0 \quad \forall \quad \bar{s}_i \neq 0, \tag{5}$$

we can use the adjusted update law

$$\dot{s}_i = f(d_i)\,[1 + \alpha\,(\beta\tilde{s}_i - 1) + g(\bar{s}_i)] + \alpha, \tag{6}$$

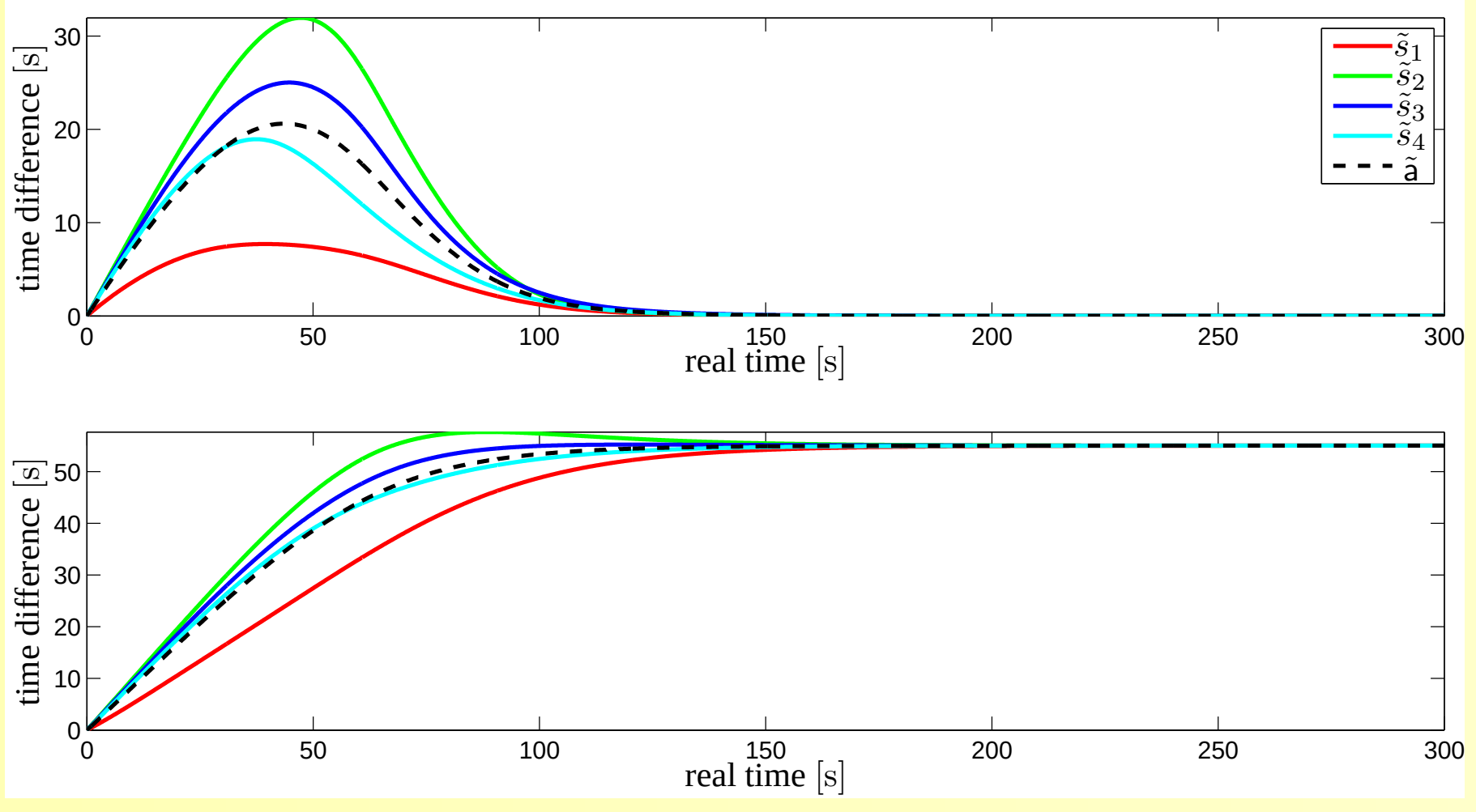
to guarantee convergence to the desired formation structure.

Path Manoeuvring

The virtual-time dynamics (2) are dependent on the distance d through $f(d)$. Here we consider the behaviour at the extremes $d \to \infty$ and $d = 0$, which will reveal the role of tuning parameters α and β:

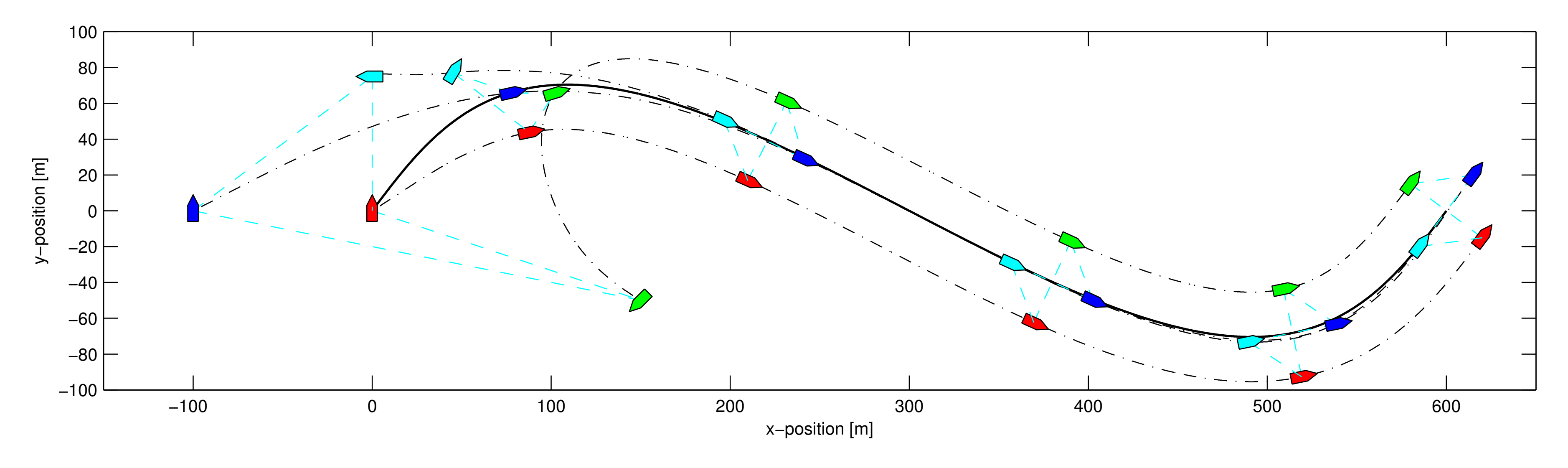
$$\begin{aligned} d \to \infty: &\quad \dot{s} = \alpha & \dot{\tilde{s}} &= 1 - \alpha \\ d = 0: &\quad \dot{s} = 1 + \alpha\beta & \dot{\tilde{s}} &= -\alpha\beta \end{aligned}$$

By changing α we choose for **path following** ($\alpha = 0$), **path manoeuvring** ($0 < \alpha < 1$), or **trajectory tracking** ($\alpha = 1$). Parameter β determines the **rate of convergence** of the time-difference $\tilde{s}$ to zero.

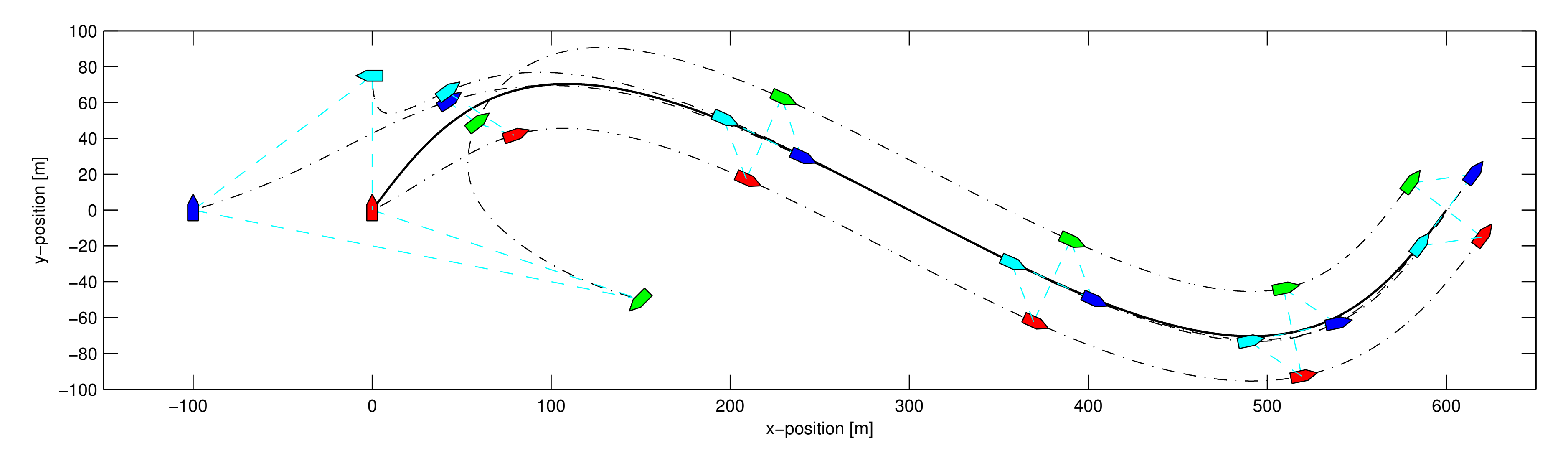


ERROR $\tilde{s}$ FOR $\alpha = 0.1$ (TOP) AND $\alpha = 0.0$

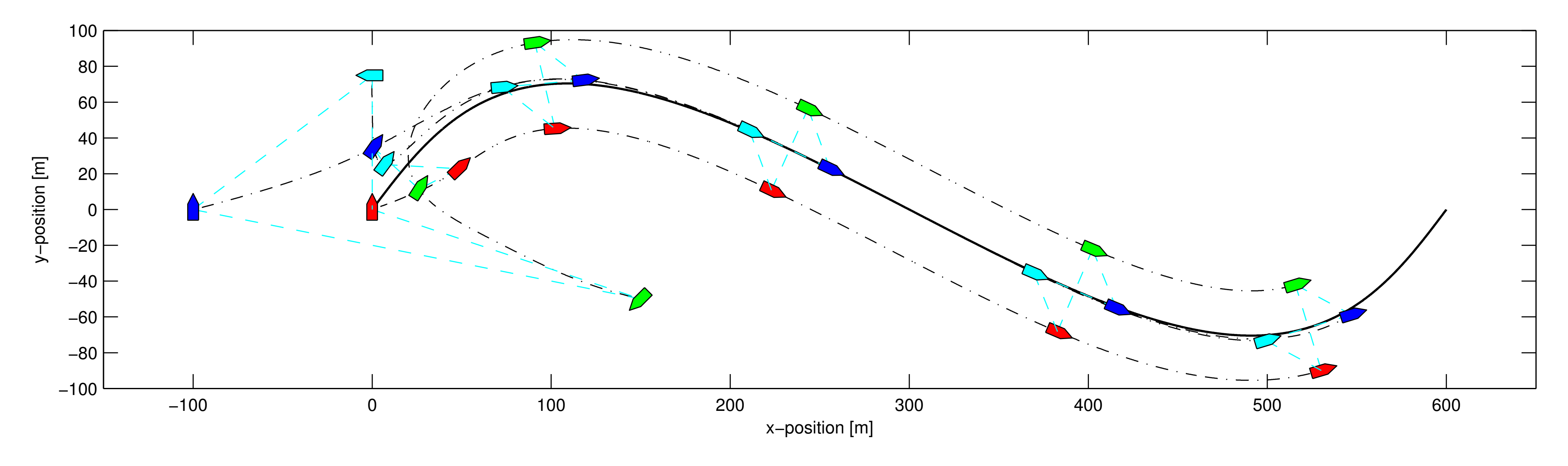
Simulation Results



TARGET TRACKING ($\alpha = 1.0$) OF THE FORMATION



PATH MANOEUVRING ($\alpha = 0.1$) OF THE FORMATION



PATH FOLLOWING ($\alpha = 0.0$) OF THE FORMATION

Discussion and Future Work

Using the proposed method we can change a trajectory-tracking controller into a more flexible path-manoeuvring controller. The advantage of path manoeuvring is that the desired path is followed closely from the beginning (as with path following), while following the trajectory with strict timing after transients (as with trajectory tracking). Open problems for this work include adding **collision avoidance**, and taking into account **communication issues** such as *delays*, *signal losses* and *discrete time-intervals*.

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