A Covariance Function Based Approach to Networked System Identification

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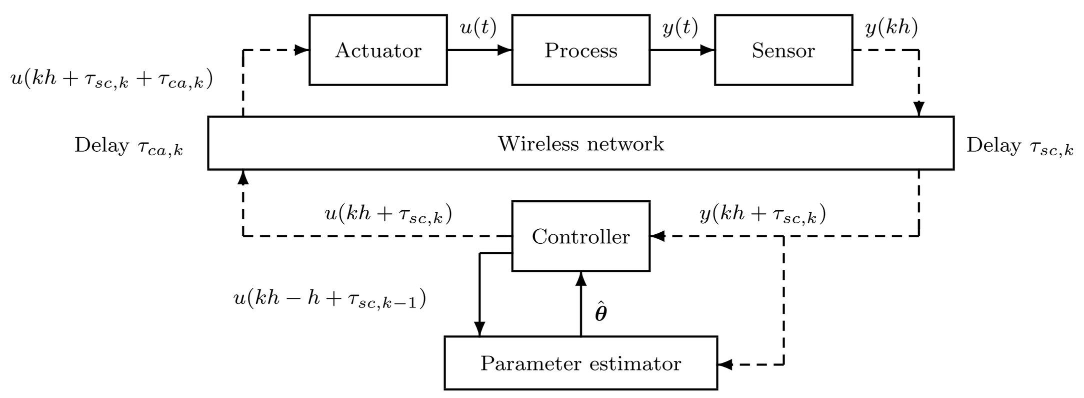
Summary

Parametric identification for networked control is considered. A covariance function based method is proposed for estimating the parameters. The method relies on the second order statistical properties of the output signal and that the input signal samples are taken from a discrete-time white noise sequence. Numerical studies indicate that the method is robust to the time-delays in the network control system, additive measurement noise and packet drop-outs.

Model description

Consider the following networked control system.



Assumptions:

1. The new input signal sample is sent to the actuator from the parameter estimator as soon as an output signal sample from the sensor arrives.
2. The input signal samples to be sent to the actuator are from a discrete-time white noise sequence.
3. The actual irregular time instants when new input signal levels are applied at the actuator are not known (and not required) at the parameter estimator.
4. If packets arrive in wrong order, the oldest packet is removed and considered lost.
5. The process is described by the continuous-time model

$$y(t) = \frac{B(p)}{A(p)}u(t)$$

where

$$A(p) = \sum_{i=0}^{n} a_i p^{n-i} = \prod_{i=1}^{n}(p-\alpha_i), \qquad B(p) = \sum_{i=1}^{m} b_i p^{m-i}$$

Furthermore, it is assumed that the system is minimum phase and that the leading coefficient $b_1 > 0$.

The corresponding model in state space form is

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t)$$
$$y(t) = \mathbf{C}\mathbf{x}(t)$$

where $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ have appropriate sizes. The parameter vector to be estimated is

$$\boldsymbol{\theta}_0 = \begin{bmatrix} a_1 & \cdots & a_n & b_1 & \cdots & b_m \end{bmatrix}$$

Discretization

Define $\tau_k = \tau_{sc,k} + \tau_{ca,k}$. The discretization for $\tau_k \leqslant 2h$ and $\tau_{k-1} \leqslant 2h$ is given by [1]

$$\mathbf{x}(kh+h) = \begin{cases} e^{\mathbf{A}h}\mathbf{x}(kh) + \int_0^{h-\tau_k} e^{\mathbf{A}\sigma}d\sigma\mathbf{B}u_k + \int_{h-\tau_k}^{h} e^{\mathbf{A}\sigma}d\sigma\mathbf{B}u_{k-1}, \\ \text{for } \tau_k \leqslant h \text{ and } \tau_{k-1} \leqslant h & \text{(A)} \\ e^{\mathbf{A}h}\mathbf{x}(kh) + \int_0^{2h-\tau_{k-1}} e^{\mathbf{A}\sigma}d\sigma\mathbf{B}u_{k-1} + \int_{2h-\tau_{k-1}}^{h} e^{\mathbf{A}\sigma}d\sigma\mathbf{B}u_{k-2}, \\ \text{for } h \leqslant \tau_k \leqslant 2h \text{ and } h \leqslant \tau_{k-1} \leqslant 2h & \text{(B)} \\ e^{\mathbf{A}h}\mathbf{x}(kh) + \int_0^{h} e^{\mathbf{A}\sigma}d\sigma\mathbf{B}u_{k-1}, \\ \text{for } h \leqslant \tau_k \leqslant 2h \text{ and } 0 \leqslant \tau_{k-1} \leqslant h & \text{(C)} \\ e^{\mathbf{A}h}\mathbf{x}(kh) + \int_0^{h-\tau_k} e^{\mathbf{A}\sigma}d\sigma\mathbf{B}u_k + \int_{h-\tau_k}^{2h-\tau_{k-1}} e^{\mathbf{A}\sigma}d\sigma\mathbf{B}u_{k-1} + \int_{2h-\tau_{k-1}}^{h} e^{\mathbf{A}\sigma}d\sigma\mathbf{B}u_{k-2}, \\ \text{for } 0 \leqslant \tau_k \leqslant h,\ h \leqslant \tau_{k-1} \leqslant 2h, \text{ and } \tau_{k-1}-\tau_k < h & \text{(D)} \\ e^{\mathbf{A}h}\mathbf{x}(kh) + \int_0^{h-\tau_k} e^{\mathbf{A}\sigma}d\sigma\mathbf{B}u_k + \int_{h-\tau_k}^{h} e^{\mathbf{A}\sigma}d\sigma\mathbf{B}u_{k-2}, \\ \text{for } 0 \leqslant \tau_k \leqslant h,\ h \leqslant \tau_{k-1} \leqslant 2h, \text{ and } \tau_{k-1}-\tau_k \geqslant h & \text{(E)} \end{cases}$$

The structure of the discrete-time model as well as the parameters describing the dependency on the values of the applied input signal are time-varying.

Estimation

Estimation of $\{a_i\}_{i=1}^n$

Define $\{c_i\}_{i=1}^n$ and $\{\gamma_i\}_{i=1}^n$ as

$$\det(q\mathbf{I} - e^{\mathbf{A}h}) = \sum_{i=0}^{n} c_i q^{n-i} = \prod_{i=1}^{n}(q-\gamma_i)$$

The relation $\alpha_i = \big(\ln(\gamma_i)\big)/h$ holds between the continuous-time and the discrete-time poles. Under Assumption 2, the covariance function $r_y(\Delta) = \mathsf{E}\{y(kh+\Delta h)y(kh)\}$ satisfies the discrete-time Yule-Walker equation

$$r_y(\Delta) + c_1 r_y(\Delta-1) + \ldots + c_n r_y(\Delta-n) = 0$$

for $\Delta \geqslant 2n$. Estimate $r_y(\Delta)$ as $\hat{r}_y(\Delta) = \frac{1}{N-\Delta}\sum_{k=1}^{N-\Delta} y(kh+\Delta h)y(kh)$ from the delayed samples at the parameter estimator. The estimates $\{\hat{c}_i\}_{i=1}^n$ are obtained from

$$\begin{bmatrix} \hat{r}_y(\Delta_1) \\ \vdots \\ \hat{r}_y(\Delta_M) \end{bmatrix} = -\begin{bmatrix} \hat{r}_y(\Delta_1-1) & \cdots & \hat{r}_y(\Delta_1-n) \\ \vdots & & \vdots \\ \hat{r}_y(\Delta_M-1) & \cdots & \hat{r}_y(\Delta_M-n) \end{bmatrix} \begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix}$$

where $M \geqslant n$. Thereafter, $\{\hat{c}_i\}_{i=1}^n \rightarrow \{\hat{\gamma}_i\}_{i=1}^n \rightarrow \{\hat{\alpha}_i\}_{i=1}^n \rightarrow \{\hat{a}_i\}_{i=1}^n$.

Estimation of $\{b_i\}_{i=1}^n$

Define the filter $z(t) = \frac{1}{A(p)}u(t)$ and $y(t) = \sum_{j=1}^{m} b_j p^{m-j} z(t)$ [2]. Then,

$$\begin{aligned} r_y(\delta) &= \mathsf{E}\{y(t+\delta)y(t)\} \\ &= \sum_{j=1}^{m}\sum_{k=1}^{m} b_j b_k (-1)^{m-k} p^{2m-j-k} r_z(\delta) \end{aligned}$$

where $r_z(\delta) = \mathsf{E}\{z(t+\delta)z(t)\}$ and $\delta \in \mathbb{R}$. The estimates $\{\hat{b}_i\}_{i=1}^n$ are obtained from

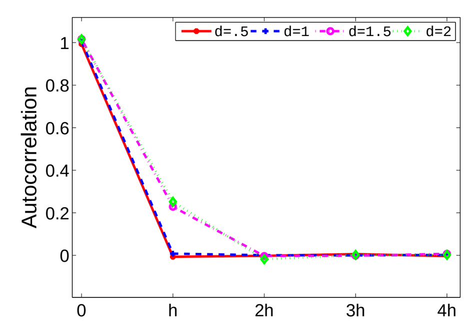
$$\begin{bmatrix} \hat{r}_y(T_1) \\ \vdots \\ \hat{r}_y(T_P) \end{bmatrix} = \begin{bmatrix} \hat{r}_z(T_1) & \cdots & (-1)^{m-1}D^{2m-2}\hat{r}_z(T_1) \\ \vdots & & \vdots \\ \hat{r}_z(T_P) & \cdots & (-1)^{m-1}D^{2m-2}\hat{r}_z(T_P) \end{bmatrix} \boldsymbol{\kappa}$$

that contains estimated covariance elements. Moreover, D is an approximation of the differentiation operator p and $\boldsymbol{\kappa} = \begin{bmatrix} b_m^2 & b_{m-1}^2 & \cdots & b_1^2 \end{bmatrix}^T$. The estimates $\{\hat{r}_z(T_P)\}_{i=1}^P$ are computed from the samples $\{\hat{z}(kh)\}_{k=1}^N$ by filtering with $1/\hat{A}(p)$, constructed from the estimates $\{\hat{a}_i\}_{i=1}^n$.

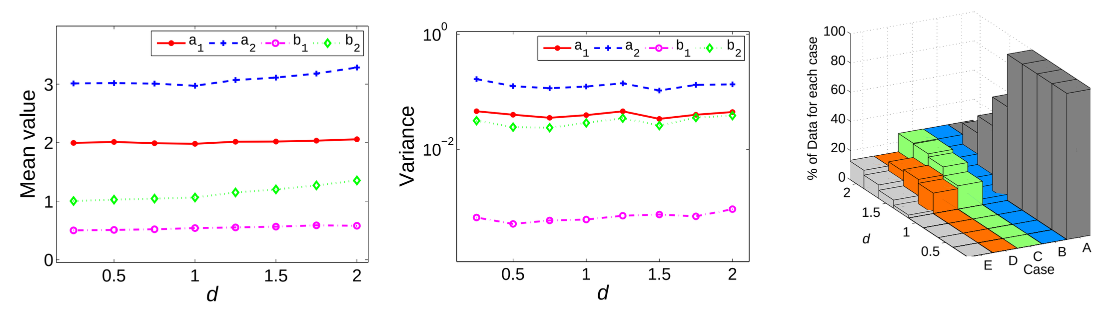
Numerical studies

Data are generated from a second order process with $a_1 = 2$, $a_2 = 3$, $b_1 = 0.5$, and $b_2 = 1$. From these data, the estimates $\hat{a}_1$, $\hat{a}_2$, $\hat{b}_1$, and $\hat{b}_2$ are computed. This procedure is carried out 200 times in different Monte Carlo simulations to obtain empirical mean values and variances.

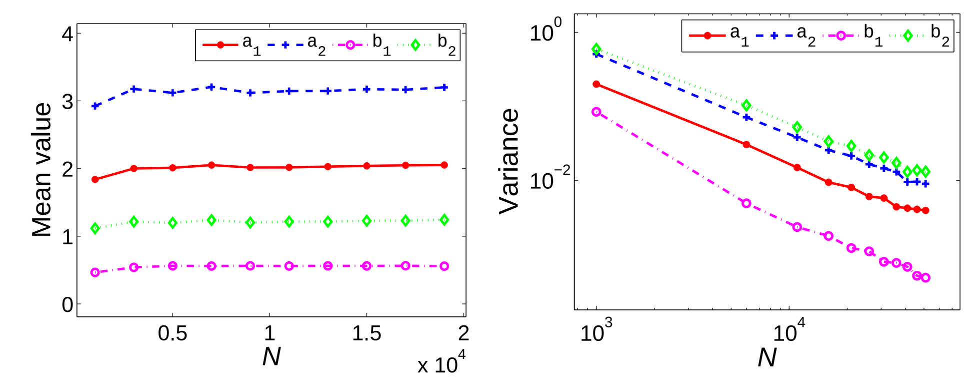
Example 1 *The degree of whiteness at the actuator.* The time-delays are uniform distribution as $\{\tau_k\} \in \mathcal{U}(0, dh)$, where $d = [0, 2]$.



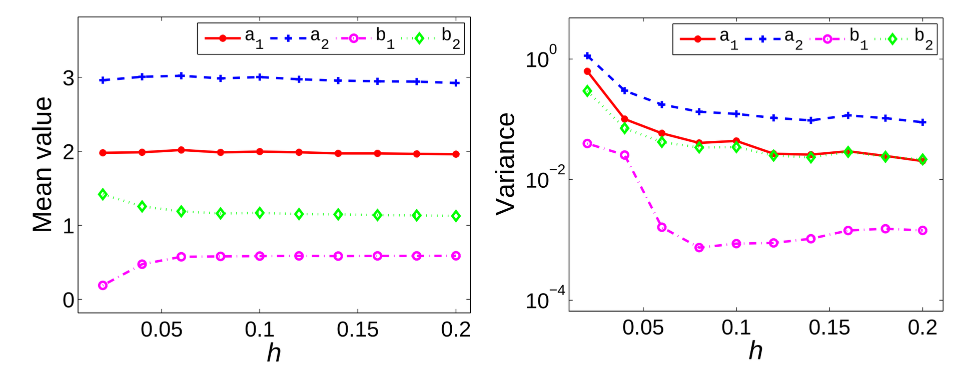
Example 2 *Influence of* τ_k. The influence of the time-delays $\{\tau_k\} \in \mathcal{U}(0, dh)$ on the estimates where $d \in [0, 2]$. The percentage of discretized data from each case (A)–(E) is also given.



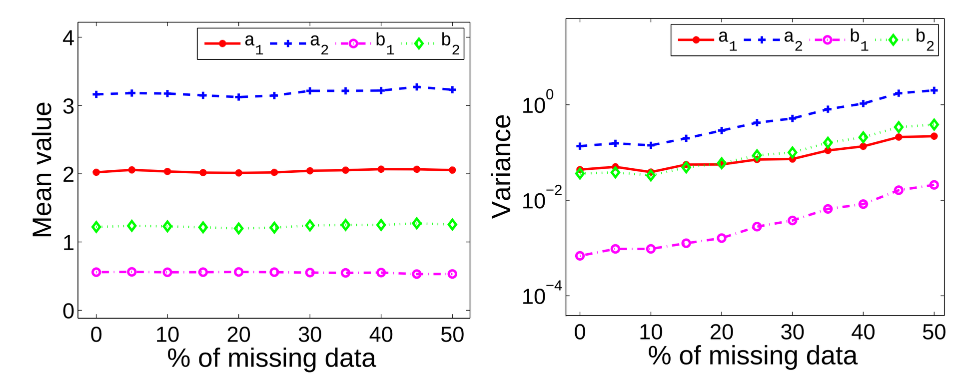
Example 3 *Influence of* N.



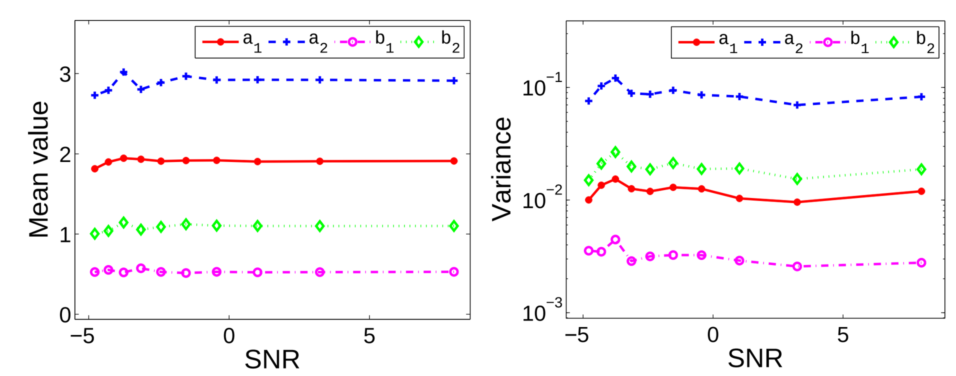
Example 4 *Influence of* h.



Example 5 *Packet drop-outs.* The percentage of output signal sample packet drop-outs at the parameter estimator is varied in the interval $[0, 50]$.



Example 6 *Sampling noise.* The SNR of the output signal samples received at the parameter estimator is varied in the interval $[-5, 8]$ dB.



References

[1] M. B. G. Cloosterman, N. van de Wouw, W. P. M. H. Heemels, and H. Nijmeijer. Stability of networked control systems with uncertain time-varying delays. *IEEE Trans. on Automatic Control*, 54(7):1575–1580, July 2009.

[2] M. Mossberg and T. Söderström. Continuous-time errors-in-variables system identification through covariance matching without input signal modeling. In *Proc. American Control Conf.*, pages 4390–4391, St. Louis, MO, June 10–12 2009.

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