Université Catholique de Louvain
Universidad de Sevilla

Energy-Aware Consensus Algorithms in Networked Sampled Systems

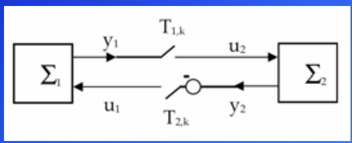
M.López-Martínez, J.-C.Delvenne and Vincent D. Blondel

ABSTRACT:

This work presents a method to analyze the convergence to consensus of a network of first order linear systems, when the signals associated to the interconnections are sampled from the continuous time systems. In order to minimize the energy consumed in the process of communication, we will look for the optimal sampling time such that the consensus is reached in a minimum number of iterations (communications). The analysis is performed by minimizing several objective functions that take into account a measure of the convergence rate to reach a consensus. These objective functions mainly depend on the eigenvalues of the sampled transition matrix of the system. Finally, we present a case study based on the torus topology, where a simple case of communication is analyzed and the optimal sampling time to reach a consensus is obtained.

MOTIVATING EXAMPLE:

- Feedback interconection of two sampled first order systems



- Closed Loop Sampled Dynamics

$$x_{k+1} = A_k x_k, \qquad A_k = \begin{bmatrix} a & b \\ b & a \end{bmatrix} \qquad \begin{aligned} a &= e^{-T_k} \\ b &= 1-a \end{aligned}$$

- Eigenvalues

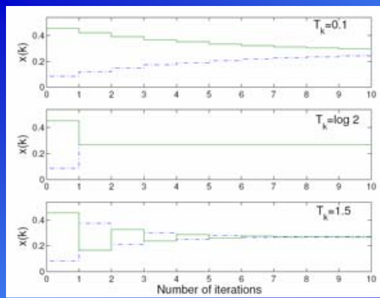
$$\begin{aligned} z_1 &= 1 \\ z_2 &= 2a - 1 = 2e^{-T_k} - 1 \end{aligned}$$

- Optimal sampling time to reduce the number of iterations

$$z_2 = 0 \longrightarrow T_k = \log 2$$

SIMULATION: TWO AGENTS

- Comparison of the number of iterations as a function of T_k



- In this case the optimal sampling time to reduce the number of iterations, and hence the energy consumed in the communication, is given by $T_k = \log 2$. It is also interesting to note that the number of iterations to reach the consensus (within a prescribed error) for $T_k = 0.1$ is larger than for $T_k = 1.5$

CONSENSUS OF SAMPLED SYSTEMS

- Consider a critically-stable continuous time interconnection of N first order linear systems.

$$\Sigma_i : \quad \dot{x}_i = -x_i + u_i \qquad u_i = \sum_j k_{ij} x_j, \quad j \neq i \in [1, N]$$

- The signals of each system are sampled and hold (ZOH) before exchanging the information.Then, the dynamics of each system with $a_i = e^{-T_k}$ yields

$$\Sigma_{i,k} : \; x_{i,k+1} = a_i x_{i,k} + (1 - a_i) \sum_j k_{ij} x_{j,k}, \quad j \neq i \in [1, N]$$

- The dynamics of the whole system are given by

$$x_{k+1} = (aI + (1-a)K) x_k = \bar{A}_k x_k \qquad \sum_{i=1}^{n} k_{ij} = 1 \quad \forall j, \quad \sum_{j=1}^{n} k_{ij} = 1 \quad \forall i$$

where K is the adjacency matrix excluding the elements of the main diagonal, which are set to zero, verifying that the matrix is doubly stochastic.

- The eigenvalues of the transition matrix can be calculated with a very simple expression, where the eigenvalues of K have to be computed just ONCE, and depend linearly on the parameter $a_i = e^{-T_k}$.

$$z(\bar{A}_k) = z(aI + (1-a)K) = a\mathbf{1} + (1-a) z(K)$$

- The optimal sampling time Tk can be obtained by minimizing several criteria:

Ex1) Second eigenvalue criteria.

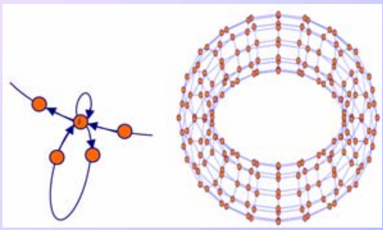
$$T^* = \arg\min_{T_k > 0} \{J_1\} = \arg\min_{T_k > 0} \{|z_2(\bar{A}_k)|\}$$

Ex2) Sum of mean quadratic errors

$$J_6 = \sum_{i \neq 1} \frac{1}{1 - |z_i|^2}$$

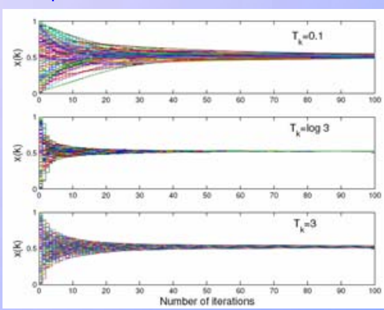
CASE STUDY: TORUS GRAPH INTERCONNECTION

- One hundred agents interconnected in a torus graph



Every agent is communicating with itself and with an adjacent agent in the same ring. (in-degree=out-degree=3)

- Comparison of the number of iterations as a function of T_k



For this case, we obtain that the optimal sampling time that minimizes the indexes J_1 and J_6 is the same, and it is given by $T^* = \log 3$.

The figure shows that the number of iterations to achieve the consensus with sampling time $T_k = \log 3$ and $T_k = 3$ is less than with sampling time $T_k = 0.1$ sec. This implies that there is a range of sampling times where the network has low energy consumption (less iterations) and requires low bandwidth to reach a consensus.

4. CONCLUSIONS

This work has shown the importance of choosing an appropriate sampling time when using consensus algorithms in networked sampled systems. The implications of this sampling time selection are mainly two, reduction of the energy consumption and reduction of bandwidth requirements. The main drawback that may appear is that these reductions may be achieved at the expense of increasing the time of convergence (in seconds) of the consensus algorithm, which will have to be also taken into account depending on the applications. For that reason, our future development will be to define and study new indexes to minimize a weighted relation between both, like for example could be $J = J_2(1 + rT_k)$, where $J_2 = -3 \log |z_2|$, is used to minimize the number of iterations and $T_k J_2$ is introduced to minimize the settling time.