Guiding one-dimensional formations with coarsely quantized information

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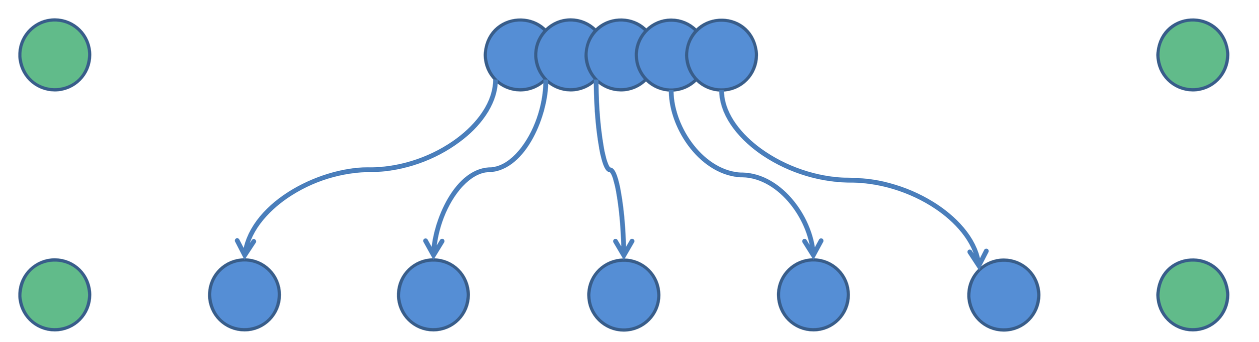
Introduction

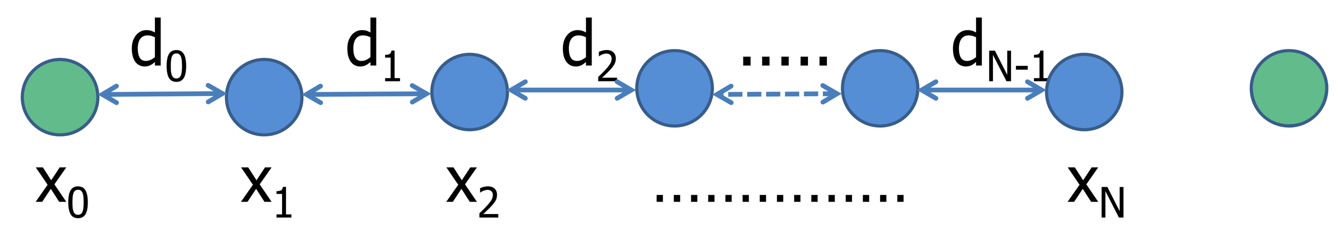
We study the deployment problem on the line for platoons of mobile agents that are guided by coarsely quantized information. Using tools from non-smooth analysis, it is shown that the formation can converge within finite time even when each agent is constrained by a 1-bit transmission channel.

Deployment problem

In a deployment problem on the line, agents must be (uniformly) positioned between the two ends of a segment



Each agent measures the distance between itself and the neighbor(s), hence the communication graph is a chain graph. We consider the case in which the last edge is not present



Choose the disagreement function

$$V(x) = \frac{1}{2}\sum_{j=0}^{N-1}(x_{j+1}-x_j-d_j)^2$$

and define the gradient system $\dot{x} = -\nabla V(x)$:

$$\begin{aligned}
\dot{x}_1 &= -(x_1-x_0-d_0)+(x_2-x_1-d_1)\\
\dot{x}_i &= -(x_i-x_{i-1}-d_{i-1})+(x_{i+1}-x_i-d_i)\\
&\quad i=2,\dots,N-1\\
\dot{x}_N &= -(x_N-x_{N-1}-d_{n-1})
\end{aligned}$$

where x_0=*const.* The agents converge to the desired configuration.

Deployment under binary information

(1) Sensors are very coarse: They measure whether inter-agent distance exceeds or is inferior to the desired one. 1 bit of information is enough.
(2) Simple control directives: Move closer if too far and move away if too close

The new model under very coarse information:

$$\begin{aligned}
\dot{x}_1 &= -k_{10}\mathrm{sgn}(x_1-x_0-d_0)+k_{11}\mathrm{sgn}(x_2-x_1-d_1)\\
\dot{x}_i &= -k_{i,i-1}\mathrm{sgn}(x_i-x_{i-1}-d_{i-1})+k_{i,i}\mathrm{sgn}(x_{i+1}-\\
&\quad x_i-d_i),\quad i=2,\dots,N-1\\
\dot{x}_N &= -k_{N,N-1}\mathrm{sgn}(x_N-x_{N-1}-d_{N-1})
\end{aligned}$$

An interpretation

If Agent i is far from i+1 and far from i-1 then it doesn't move
If Agent i is close to i+1 and far from i-1 then it moves towards i-1 and away from i+1

Analysis

The system above is discontinuous and solutions are Krasowskii solutions. i.e. $\dot{x} \in \mathcal{K}(g(t,x))$ where

$$\mathcal{K}(g(t,x)) = \bigcap_{\delta>0}\overline{\mathrm{co}}\,(g(t,B(x,\delta)))$$

The analysis is better performed on the system expressed in the coordinates $z_i = x_{i+1}-x_i$, which give

$$\begin{aligned}
\dot{z}_0 &= -k_{10}\mathrm{sgn}(z_0-d_0)+k_{11}\mathrm{sgn}(z_1-d_1)\\
\dot{z}_i &= k_{i,i-1}\mathrm{sgn}(z_{i-1}-d_{i-1})-(k_{i+1,i}+k_{ii})\mathrm{sgn}(z_i-d_i)\\
&\quad +k_{i+1,i+1}\mathrm{sgn}(z_{i+1}-d_{i+1}),\ i=1,2,\dots,N-2\\
\dot{z}_{N-1} &= k_{N-1,N-2}\mathrm{sgn}(z_{N-2}-d_{N-2})-(k_{N,N-1}\\
&\quad +k_{N-1,N-1})\mathrm{sgn}(z_{N-1}-d_{N-1})
\end{aligned}$$

For this system, the following holds:

Theorem

For any ε>0, let

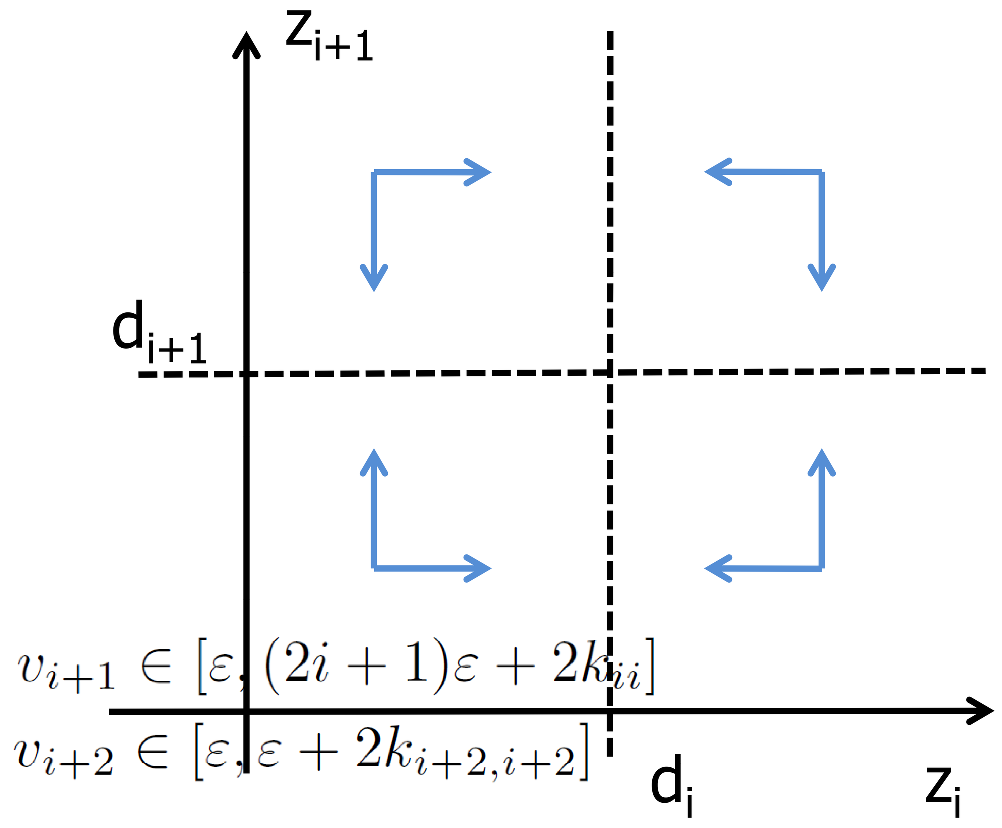
$$\begin{aligned}
k_{i,i-1}-k_{i,i} &= i\varepsilon\,, \quad i=1,\dots,N-1\\
k_{N,N-1} &= N\varepsilon\,.
\end{aligned}$$

Then the equilibrium $z_* := (d_0,d_1,\dots,d_{N-1})$ is GAS.

Trajectory-based analysis

The focus is on the sub-system

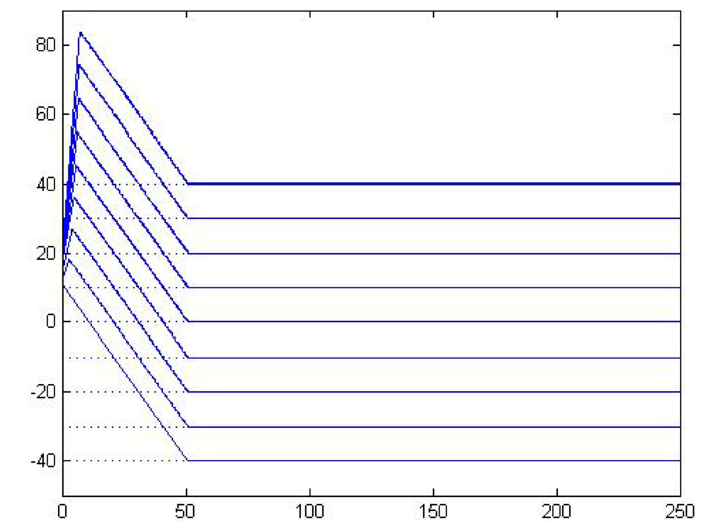
$$\begin{aligned}
\dot{z}_i &= k_{i,i-1}\mathrm{sgn}(z_{i-1}-d_{i-1})-(k_{i+1,i}+k_{ii})\\
&\quad \mathrm{sgn}(z_i-d_i)+k_{i+1,i+1}\mathrm{sgn}(z_{i+1}-d_{i+1})\\
\dot{z}_{i+1} &= k_{i+1,i}\mathrm{sgn}(z_i-d_i)-(k_{i+2,i+1}+k_{i+1,i+1})\\
&\quad \mathrm{sgn}(z_{i+1}-d_{i+1})+k_{i+2,i+2}\mathrm{sgn}(z_{i+2}-d_{i+2})
\end{aligned}$$

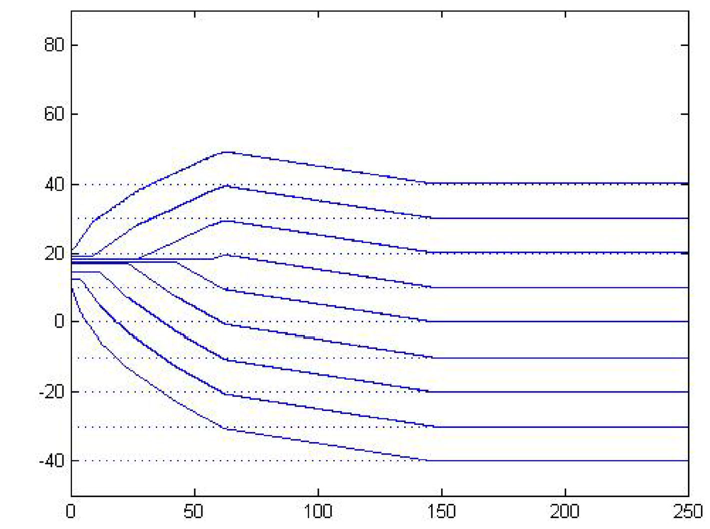


Exploiting that $v \in \mathcal{K}(f(z))$ implies $v \in \times_{i=0}^{N-1}\mathcal{K}(f_i(z))$ we can analyze the behavior of the solutions studying $v_{i+1} \in \mathcal{K}(f_{i+1}(z))$ and $v_{i+2} \in \mathcal{K}(f_{i+2}(z))$. Convergence to the axes occurs in finite time with velocity ε. A sliding mode occurs on the axes.

Numerical results

Deployment within the segment [-50,50] of N=9 agents with ε=1 and tight initial positions





Conclusions

Complex coordination tasks (deployment) can be achieved using binary measurements and simple control commands. Chattering can be avoided using hysteresis.