Distributed Estimation and Detection via Local Information

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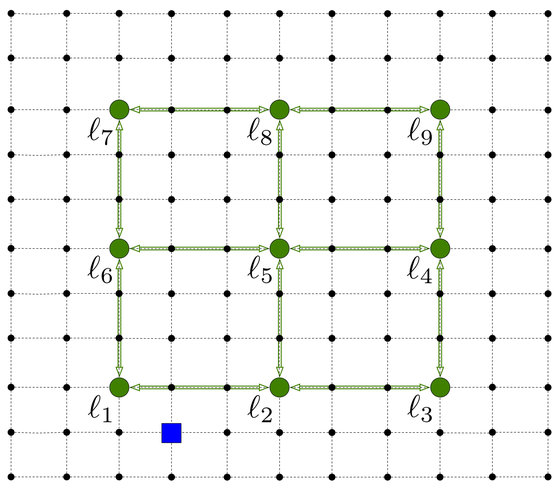
Multi-agent system

Network of autonomous agents able to sense, communicate, and process information

- rendezvous
- formation
- clock synchronization
- load balancing

Model description



- A vertex in the graph corresponds to an agent
- linear interaction with neighboring agents

$$\mathbf{x(t+1)} = \mathbf{Ax(t)} + \mathbf{v(t)}$$

- $\mathbb{E}[v(t)] = 0$, $\mathbb{E}[v(t)v(\tau)^T] = \sigma_v \delta(t-\tau) I$
- a *leader* (●) has better computation and communication capabilities
- a *leader-graph* connects the leaders
- a *misbehaving agent* (■) deviates from the nominal behavior

$$\mathbf{x(t+1)} = \mathbf{Ax(t)} + \mathbf{Eu(t)} + \mathbf{v(t)}$$

- the misbehaving input u is arbitrary

Local observation

Each leader ℓ_i observes directly the state of its neighbors

$$\mathbf{y}_{\ell_i}\mathbf{(t)} = \mathbf{C}_{\ell_i}\mathbf{x(t)} + \mathbf{w}_{\ell_i}\mathbf{(t)}$$

- $\mathbb{E}[w_{\ell_i}(t)] = 0$, $\mathbb{E}[w_{\ell_i}(t) w_{\ell_i}^T(\tau)] = \sigma_v \delta(t-\tau) I$
- $\mathbb{E}[v(t) w_{\ell_i}(t)] = 0$, $\mathbb{E}[w_{\ell_i}(t) w_{\ell_j}^T(t)] = 0$
- each row of C_{ℓ_i} has one nonzero element

Problem definition

Assumptions:

- each leader has local network knowledge
- leaders cooperate

Estimation: Compute an optimal estimate of (a linear function of) the network initial state

Detection: Detect the presence of a misbehaving agent from local measurements

Estimation problem

Let each leader collect $s \in \mathbb{N}$ measurements

$$\mathbf{Y^s} = \mathbf{O^s x}(\mathbf{0}) + \mathbf{F^s U^s}$$

where

$$Y^s = \begin{bmatrix} Y_1^s \\ Y_2^s \\ \vdots \\ Y_m^s \end{bmatrix} \quad O^s = \begin{bmatrix} O_1^s \\ O_2^s \\ \vdots \\ O_m^s \end{bmatrix} \quad F^s = \begin{bmatrix} F_1^s \\ F_2^s \\ \vdots \\ F_m^s \end{bmatrix}$$

$$O_i^s = \begin{bmatrix} C_{\ell_i} \\ C_{\ell_i}A \\ C_{\ell_i}A^2 \\ \vdots \\ C_{\ell_i}A^{s-1} \end{bmatrix} \quad Y_i^s = \begin{bmatrix} y_{\ell_i}(0) \\ y_{\ell_i}(1) \\ y_{\ell_i}(2) \\ \vdots \\ y_{\ell_i}(s-1) \end{bmatrix}$$

$$F_i^s = \begin{bmatrix} D_{\ell_i} & 0 & \cdots & \cdots & 0 \\ C_{\ell_i}B & D_{\ell_i} & \ddots & \ddots & 0 \\ C_{\ell_i}AB & C_{\ell_i}B & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ C_{\ell_i}A^{s-2}B & C_{\ell_i}A^{s-3}B & \cdots & C_{\ell_i}B & D_{\ell_i} \end{bmatrix}$$

Estimation method

The minimum variance estimate of $x(0)$ is

$$\mathbf{x}_{\text{wls}} = ((\mathbf{O^s})^\mathbf{T}\mathbf{Q^{-1}}\mathbf{O^s})^{-1}(\mathbf{O^s})^\mathbf{T}\mathbf{Q^{-1}}\mathbf{Y^s}$$

where, being Λ diagonal positive definite,

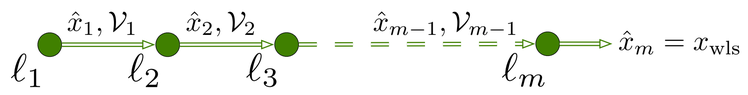
$Q = \mathbb{E}[(F^s U^s)(F^s U^s)^T] = (F^s \Lambda^{\frac{1}{2}})(\Lambda^{\frac{1}{2}}(F^s)^T)$

(Thm. 1):

$$\begin{bmatrix} x_{\text{wls}} \\ * \end{bmatrix} = \lim_{\varepsilon \to 0^+} \left[O^s \ \varepsilon F^s \Lambda^{\frac{1}{2}} \right]^\dagger Y^s$$

As ε decreases, the pseudoinverse solution approaches the minimum variance estimate

Alg. 1: Sequential pseudoinverse algorithm



Assumption:

- leader ℓ_i knows Y_i^s, O_i^s, $\varepsilon F_i^s \Lambda^{\frac{1}{2}}$

During the i-th iteration, leader ℓ_i

1. receives $\hat{x}_{i-1}$ and $\mathcal{V}_{i-1} = \cap_{j=1}^{i-1} \text{Ker}([O_j^s \ F_j^s]$
2. updates $\hat{x}_{i-1}$ along $\mathcal{V}_{i-1}$ to satisfy its equations
3. transmits $\hat{x}_i$ and $\mathcal{V}_i = \mathcal{V}_{i-1} \cap \text{Ker}([O_i^s \ F_i^s])$

Alg. 2: Sequential estimation algorithm

Input : Y_i^s, O_i^s, $\varepsilon F_i^s \Lambda^{1/2}$;

if $i = 1$ **then** $\hat{x} := 0$ and $K := I$;

else receive $\hat{x}$ and K from leader ℓ_{i-1};

$\hat{x} := \hat{x} + K([O_i^s \ \varepsilon F_i^s \Lambda^{1/2}]K)^\dagger (Y_i^s - [O_i^s \ \varepsilon F_i^s \Lambda^{1/2}]\hat{x})$;

$K := \text{Basis}(K \, \text{Ker}([O_i^s \ \varepsilon F_i^s \Lambda^{1/2}]K))$;

if $i < m$ **then** transmit $\hat{x}$ and K to leader ℓ_{i+1};

else transmit $\hat{x}$ to every leader;

return $\hat{x}$;

Detection method

A misbehavior affect the input-output relation

$$\mathbf{Y^s} = \mathbf{O^s x} + \mathbf{F^s U^s} + \mathbf{E^s U_e^s}$$

If $E^s U_e^s = 0$, the estimation error satisfies *(Thm. 2):*

$$\lim_{\varepsilon \to 0^+} \| Y^s - O^s x_{\text{wls}} \| \leq \| I - O^s W \| \| U^s \|$$

where $\mathbf{W} = \left((\mathbf{O^s})^\mathbf{T}\mathbf{Q^{-1}}\mathbf{O^s}\right)^{-1}(\mathbf{O^s})^\mathbf{T}\mathbf{Q^{-1}}$

A misbehavior is detected if the residual is greater than a predetermined threshold

Alg. 3: Sequential detection algorithm

Input : Y_i^s, O_i^s, $\varepsilon F_i^s \Lambda^{1/2}$, γ;

while *True* **do**

collect the measurements $Y_{i,t}^s$;

execute *Sequential estimation algorithm*;

if $\| Y_{i,t}^s - O_i^s \hat{x} \|_\infty > \gamma$ **then**

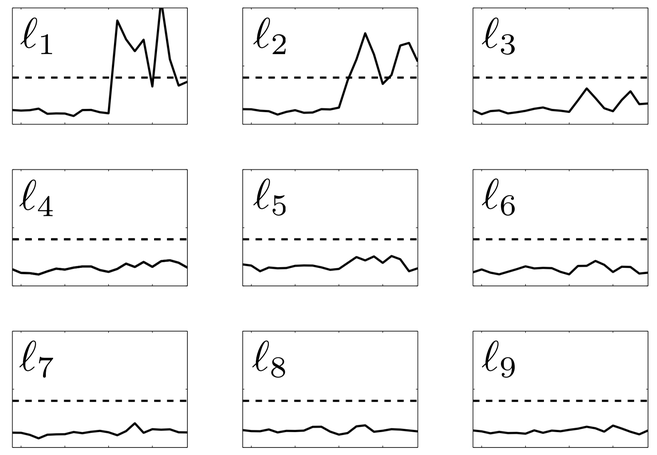
return *Anomaly;*

Detection of a misbehaving agent

The misbehaving signal is $e(t) = \bar{e} + e_w(t)$

- $\mathbb{E}[e_w(t)] = 0$, $\mathbb{E}[e_w(t) e_w(\tau)^T] = 0$

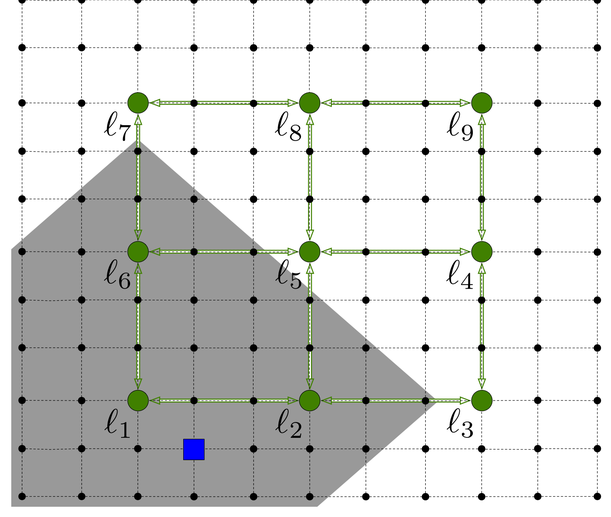
Residual functions and decision threshold:



- regional detection via residual analysis

Remarks and ongoing research

The computation of the matrices O_i^s and $F_i^s \Lambda^{\frac{1}{2}}$ requires only local knowledge of the network



Future directions:

- finite memory estimation algorithm
- detection method for unbounded noise