Stability Analysis of Networked Control Systems Subjected to Packet-dropouts and Finite Level Quantization

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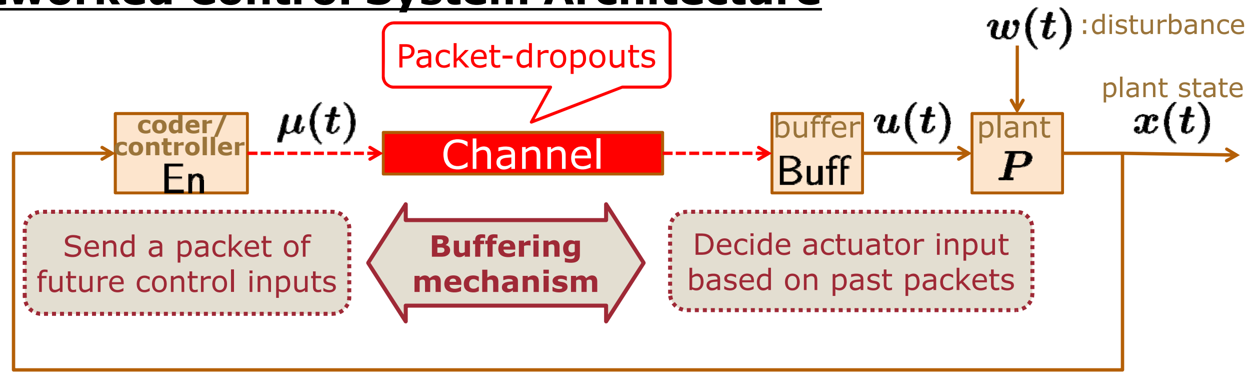
Networked Control System Architecture



Plant P: Linear Time-Invariant system

$$x(t+1) = Ax(t) + Bu(t) + w(t), \quad x(0) = 0.$$

Channel Affected by packet-dropouts

Consecutive packet-dropouts are bounded!!

$$1 \le t_{i+1} - t_i \le N \qquad \forall i \in \mathbb{Z}_+$$

(t_i : Time steps when transmission succeeds)

Buffer Buff : Outputs the top of its memory

$$u(t) = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} b(t),$$

Memory is updated based on received packets:

$$b(t) = s(t)\mu(t) + (1 - s(t)) \begin{bmatrix} 0 & 1 & \cdots & 0 \\ & \ddots & \ddots & \vdots \\ & & \ddots & 1 \\ 0 & & & 0 \end{bmatrix} b(t-1),$$

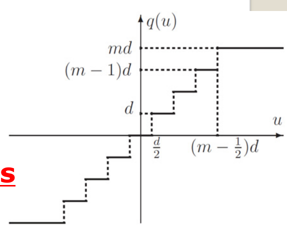
$$s(t) = \begin{cases} 1 & \text{if transmission succeeds,} \\ 0 & \text{if transmission fails.} \end{cases}$$

Controller-Encoder En: Predicts future N-1 steps, Sends a packet of possible quantized control inputs.

$$\mu(t) = \begin{bmatrix} q(\hat{u}(t;t)) \\ q(\hat{u}(t+1;t)) \\ \vdots \\ q(\hat{u}(t+N-1;t)) \end{bmatrix}$$

: control packet

Nearest-type quantizer q :



$$M := 2m + 1$$

: Number of quantization levels

d : **Fineness of quantization**

: quantization parameters

Prediction

$$\hat{x}(t+i;t) = \begin{cases} x(t), & \text{if } i = 0, \\ A\hat{x}(t+i-1;t) + Bq(\hat{u}(t+i-1;t)), & \text{if } i = \{1, \cdots, N-1\}. \end{cases}$$

: i-th step prediction of plant state

$$\hat{u}(t+i;t) = K\hat{x}(t+i;t) \qquad (A + BK : \text{stable})$$

Problem Formulation

Find a condition for small ℓ^∞ signal ℓ^∞ stability

Let P, K given.
Find a condition on the parameters M, d, N for the small ℓ^∞ signal ℓ^∞ stability of the closed-map from w to x, i.e., for the existence of positive constants ϵ, γ s.t.

$$\|w\|_{\ell^\infty} \le \epsilon \Rightarrow \|x\|_{\ell^\infty} \le \gamma\epsilon$$

Sufficient Condition for Stability

$$M > \gamma_D + \gamma_G$$

Number of quantization levels should be large enough

$$\gamma_D := \sum_{j=0}^{N-2} |K A_K^j B|$$

γ_G : ℓ^∞-induced norm of an LTV system G_{sc}

Computation of γ_G

LMI-based method for estimating γ_G is given.

Example: Scalar Plant Case

P : $x(t+1) = ax(t) + u(t) + w(t), \quad a > 1$

$K = -a$: stabilizing feedback gain

Stability Condition

$$M > \underbrace{a}_{\gamma_D} + \underbrace{a\sqrt{a^{2(N-1)} + a^{2(N-2)} + \cdots + a^2 + 2}}_{\text{Upper bound on } \gamma_G}$$

The number of quantization level should be taken large if

- the degree of unstability of plant is large.
- the maximum number of consecutive packet-dropouts is large.

THANK YOU FOR YOUR INTEREST!