A generalized utility maximization problem with outage constraints in CDMA networks

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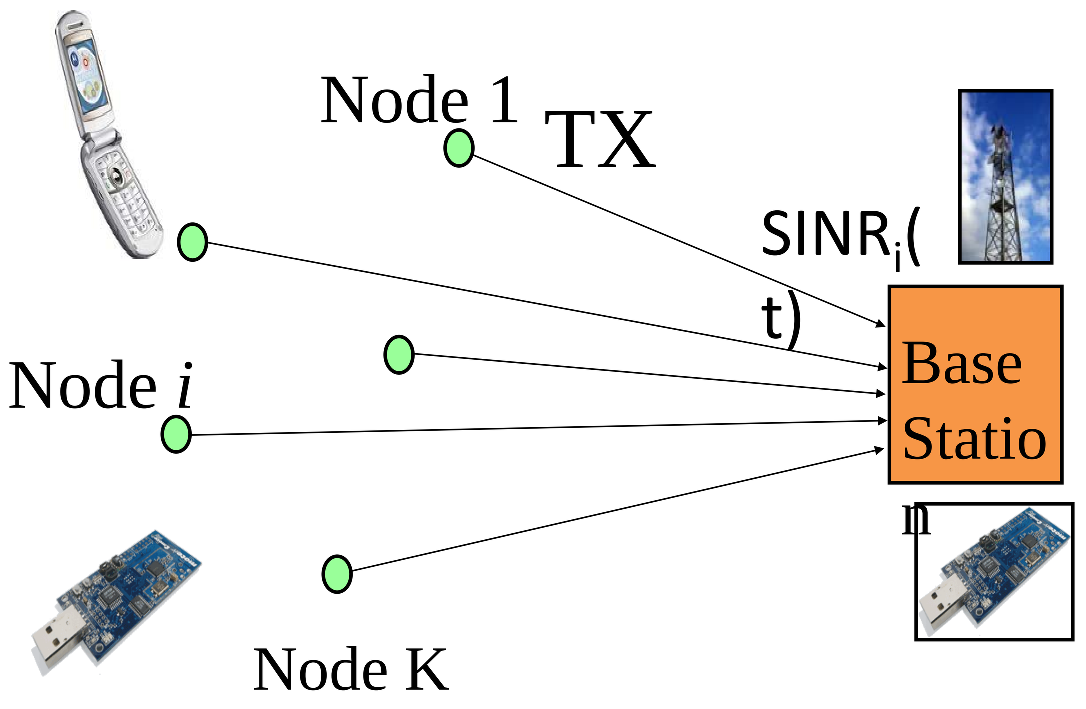
System model and utility maximization problem



Outage constraint

$$\max_{\mathbf{p},\mathbf{n}} f(\mathbf{n},\mathbf{p})$$

$$\text{s.t.}\quad \Pr\left[\mathrm{SINR}_i(\mathbf{n},\mathbf{p}) < \gamma_i\right] \leq P_{\text{out}}\,,\quad \forall i=1,\dots,K$$

$$\mathbf{p}^T(\mathbf{n}\circ\mathbb{E}[\mathbf{h}]) \leq P_T\,,$$

$$p_{i0} \leq p_i \leq p_{\max}\,,\qquad \forall i=1,\dots,K$$

$$1 \leq n_i \leq G_{i0},\qquad n_i\in\mathcal{N},\qquad \forall i=1,\dots,K$$

$\mathbf{n},\mathbf{p}$ transmit rates and radio powers

Both sigmoidal cost function and monotonic cost functions are considered

$$f(\mathbf{n},\mathbf{p}) = \sum_{i=1}^{K} n_i(1-\mathrm{BER}(\mathrm{SINR}_i(\mathbf{n},\mathbf{p})))\qquad\qquad f(\mathbf{n},\mathbf{p}) = \boldsymbol{\beta}^T\mathbf{n} - \boldsymbol{\vartheta}^T\mathbf{p}$$

Solving the utility maximization problem

***Theorem** 1:* Let the problem be feasible. Then,

- if the cost function is sigmoidal, the optimal solution $\mathbf{p}^*(\mathbf{n})$ is given by the solution to the following system of $2K+1$ equations in the variables $\mathbf{p}$ and $\boldsymbol{\lambda} = (\lambda_1,\dots,\lambda_K,\lambda_{K+1})$:

$$\nabla f(\mathbf{n},\mathbf{p}^*) - \sum_{i=1}^{K}\lambda_i(1-\nabla I_i^{(2)}(\mathbf{n},\mathbf{p}^*)) + \lambda_{K+1}\nabla\mathbf{p}^{*T}(\mathbf{n}\circ\mathbb{E}[\mathbf{h}]) = 0$$

$$\lambda_{K+1}\nabla\mathbf{p}^{*T}(\mathbf{n}\circ\mathbb{E}[\mathbf{h}]) = 0$$

  where ∇ is the gradient operator with respect to the variable $\mathbf{p}$. The solution to this system of equation can be achieved with a complexity of $O(K^3)$.
- if the cost function is monotonic, the optimal solution $\mathbf{p}^*$ is given by the solution to the system of equations given by the constraints:

$$p_i^* = I_i^{(2)}(\mathbf{n},\mathbf{p}^*),\quad i=1,\dots,K\,.$$

***Proposition** 1:* Assume that the problem is infeasible at $\bar{\mathbf{n}}$. If $\mathbf{n}\succeq\bar{\mathbf{n}}$, then the problem is infeasible at $\mathbf{n}$.

Numerical results

Algorithm

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Initialize the rate vector;
Solve the optimization problem by Theorem 1;
Reduce the set of rates N by using Proposition 1;
for any rate vector in the remaining set of rates do
    Solve the optimization problem by Theorem 1;
    Reduce the set of rates by Proposition 1;
end for;
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