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Controller Structure Design for Decentralized Control of Coupled Higher Order Subsystems

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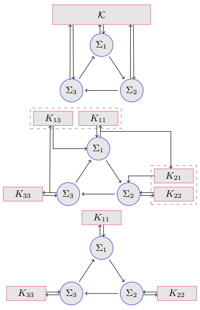
Motivation

Decentralized Control

Multiple controllers, each with access to different sensors and actors

- centralized controller
- partly decentralized controller
- fully decentralized controller

Constraints on controller topology decrease performance.



Idea: The used measurement links (i.e. the controller topology) are not specified in advance but considered as an optimization objective.

Problem Setup

Interconnected Systems

Individual systems are modeled as N LTI systems

$$G_i : \begin{cases} x_i(k+1) &= A_{ii}x_i(k) + \sum_{j\neq i} A_{ij}x_j(k) \\ & \quad +B_{1i}w_i(k) + B_{2i}u_i(k) \\ z_i(k) &= C_i x_i(k) + D_{1i}w_i(k) + D_{2i}u_i(k) \end{cases}$$

$x_i(k) \in \mathbb{R}^n$ state, w_i exogenous input, u_i control input, z_i performance output of subsystem i
$A_{ij} \neq 0$ if and only if subsystem j influences subsystem i

Controller Design

Given: Centralized state feedback controller $\mathcal{K}_{\text{opt}}$

$$u_i(k) = \sum_{j=1}^{N} K_{ij}x_j(k)$$

$$u(k) = \mathcal{K}_{\text{opt}}x(k), \text{ with } \mathcal{K}_{\text{opt}} = [K_{ij}]$$

Find a decentralized controller $\hat{\mathcal{K}} = [\hat{K}_{ij}]$ with $\hat{K}_{ij} = 0$ for as many (i,j) as possible, while the $\mathcal{H}_\infty$-performance degradation is smaller than γ w.r.t $\mathcal{K}_{\text{opt}}$.

Pattern matrix

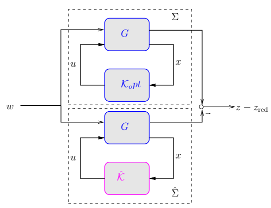
Represents the topology of the decentralized controller

$$\hat{\mathcal{Z}} := \begin{bmatrix} \|\hat{K}_{11}\|_{\ell_0} & \dots & \|\hat{K}_{1N}\|_{\ell_0} \\ \vdots & \ddots & \vdots \\ \|\hat{K}_{N1}\|_{\ell_0} & \dots & \|\hat{K}_{NN}\|_{\ell_0} \end{bmatrix}$$

Each element of $\hat{\mathcal{Z}}$ represents one block $\hat{K}_{ij}$ of the controller $\hat{\mathcal{K}}$ and $\hat{z}_{ij} = 0$ if and only if $\hat{K}_{ij} = 0$.

Error System

The decentralized controller should be 'close' to the centralized one in terms of $\mathcal{H}_\infty$-performance.



Formulation of the Optimization Problem

ℓ_0 Optimization Problem

Given the linear system G and the centralized controller $\mathcal{K}$, determine a decentralized controller $\hat{\mathcal{K}}$, such that

(i) the closed-loop system $\hat{\Sigma}$ is asymptotically stable and

(ii) the ℓ_0-norm of the pattern matrix $\hat{\mathcal{Z}}$ is minimized subject to a given maximal $\mathcal{H}_\infty$-performance degradation γ

$$\min_{\hat{\mathcal{K}} \text{ stabilizing}} \left\|\text{vec}(\hat{\mathcal{Z}})\right\|_{\ell_0}$$
$$\text{s.t. } \left\|\Sigma - \hat{\Sigma}\right\|_\infty < \gamma$$

and $\gamma > 0$ small.

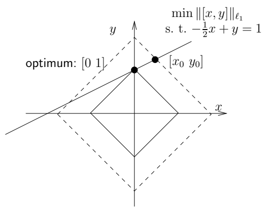
This problem is non-convex in the objective function and the constraints.

Convex Relaxation for Numerical Solution

Idea: Replace ℓ_0 by ℓ_1

Observation: ℓ_1-minimization gives sparse solutions.

- ℓ_1-norm is the convex envelope of the ℓ_0-norm.
- the ℓ_1-ball is 'pointy'.



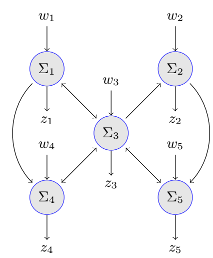
Relaxed Optimization Problem

ℓ_1 Optimization Problem

$$\min_{\hat{\mathcal{K}} \text{ stabilizing}} \left\|\text{vec}(\hat{\mathcal{Z}})\right\|_{\ell_1}, \quad \text{with } \hat{\mathcal{Z}} = [\|K_{ij}\|_{\ell_1}]$$
$$\text{s.t. } \left\|\Sigma - \hat{\Sigma}\right\|_\infty < \gamma$$

Use iLMI alg. to solve the $\mathcal{H}_\infty$-constraints (e.g. CCL).

Example



unstable system with strong couplings between individual subsystems

$$x_{k+1} = \begin{bmatrix} 1.5 & 0 & .9 & 0 & 0 \\ 0 & .5 & .1 & 0 & 0 \\ .7 & 0 & .5 & .1 & .4 \\ .2 & 0 & .1 & .8 & 0 \\ 0 & .1 & .1 & 0 & .3 \end{bmatrix} x_k$$
$$+ Iw_k + Iu_k$$
$$z_k = x_k.$$

	$\|\hat{\Sigma}\|_\infty$	$\|\hat{\mathcal{Z}}\|_{\ell_0}$
centralized controller	1.17	25
decentralized controller	1.2	14
diagonal controller	5.74	5

Topology design by ℓ_1-minimization leads to **sparse/decentralized** controllers.

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