RF-Power Overlapping Control for Connectivity Awareness in Wireless Ad-Hoc and Sensor Networks

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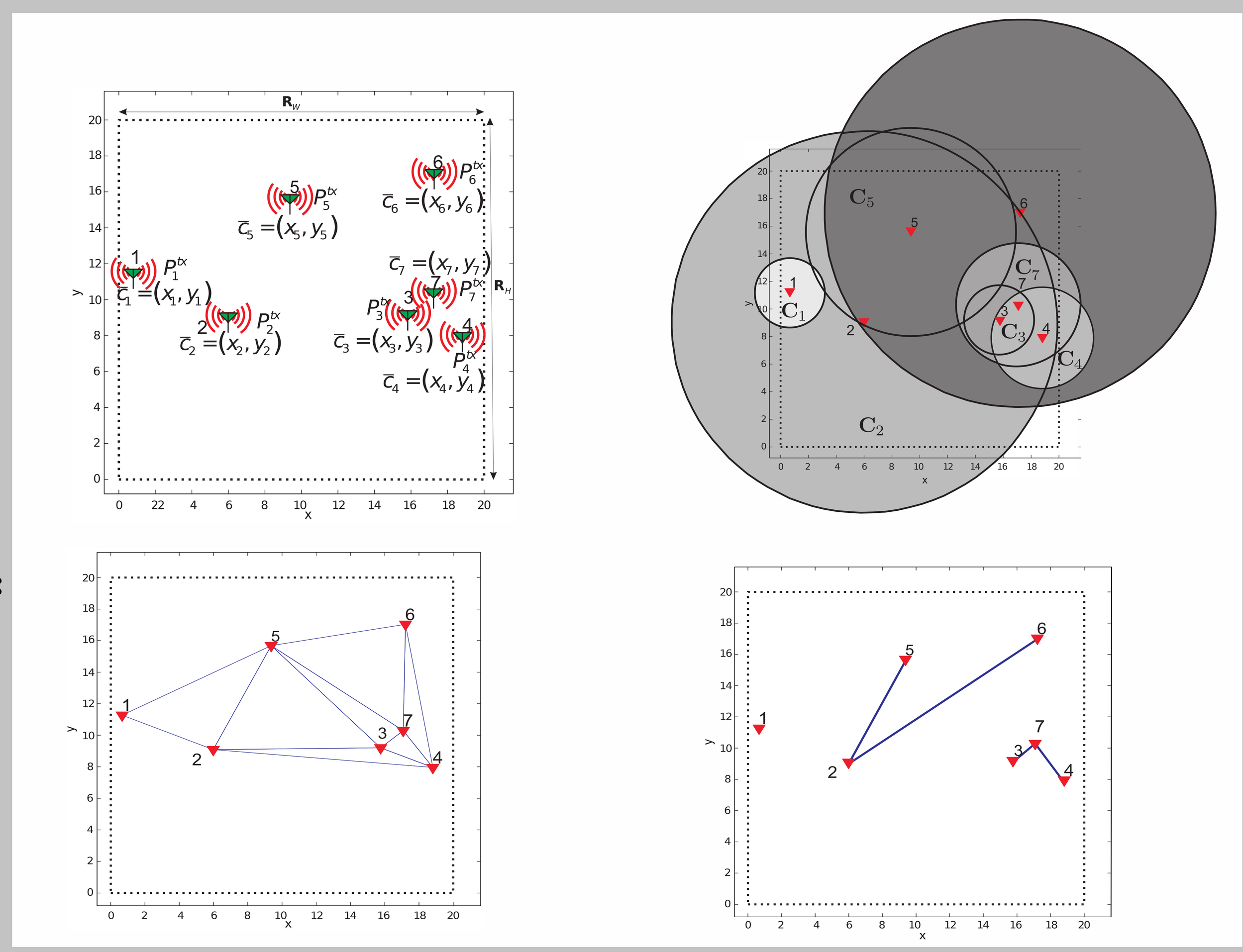
Introduction-Problem Statement

Objective: Establishment of end-to-end connectivity
Connectivity is associated to coverage: it is geometrically proven that sensing coverage can ensure connectivity, if the sensing coverage radius is at least equal to half of the network transmission radius.
In the general case:
The connectivity problem is associated to the calculation of the critical tranmsission range. Minimum transmission power is not the optimal solution from a macroscopic, power-aware point of view. However, determining the optimal transmission range remains essential, as the lower bound, upon which end-to-end connectivity is guaranteed.

The end-to-end connectivity problem is herein approached as a proximity problem (Computational Geometry, Delaunay Tessellation): Sufficient conditions for establishing end-to-end connectivity based on the Delaunay tessellation are provided and exploited for constructing the Distributed Delaunay Connectivity Algorithm.

Strengths
+ No network overhead / no need for special messages transmission
+ Distributed Delaunay Connectivity Algorithm converges in quadratic time
+ Small gap among sub-optimal and optimal condition.

Weaknesses
- A-priori awareness of local Delaunay tessellation (approximation algorithms)
- Sub-optimality



Wireless Ad-Hoc and Sensor Network (WASN) operating in a 2D, bounded region, characterized by:
(a) the spatial graph $G_{S_N} = (G_N, E_{S_N})$ and (b) the network graph $G_N = (G_N, E_N)$.
The network graph is derived from the end-to-end deterministic model (cyclic tranmission range $\mathbf{C}$), based on which the direct and indirect connectivity parameters are defined as:

Direct connectivity parameter $\alpha_{ij} = \begin{cases} 1 \text{ if } i \in \mathbf{C}_j \text{ and } j \in \mathbf{C}_i, \\ 0 \text{ otherwise.} \end{cases}$

Indirect connectivity parameter $b_{ij} = 1 - \prod_{m=1}^{|P_{ij}|-1} \alpha_{P_{ij}\{m\}P_{ij}\{m+1\}},$

where $P_{ij} = \{i, \ldots, k, \ldots, j\}$, $(i, k, \ldots, j \in \Gamma_N)$ **is a path between nodes i and j.**

End-to-end connectivity condition:

$$\sum_{i,\, j\, \in\, \Gamma_N} b_{ij} = 0$$

Distributed End-to-End Connectivity based on Delaunay Awareness

Problem 1: For the WASN comprised of N nodes, operating within a 2D bounded region R, described by graph G_{S_N} and G_N, calculate the minimum transmission power $P_i^{tx} \leq \varrho^{\max}$, $\forall i \in \Gamma_N$, such that the end-to-end network connectivity condition holds.

Lemma 1: For a WASN, whose spatial and network configuration are described by the spatial graph G_{S_N} and network graph G_N respectively, it is proven that, if a (direct or indirect) connectivity link among node $i \in \Gamma_N$ and all members $j \in \Delta_i$ is established via Delaunay edges e_{pq} $(p, q \in \{i\} \bigcup \Delta_i)$, then end-to-end connectivity among all members of Γ_N is established.
Equivalently, it is proven that if:

$$\sum_{j \in \Delta_i} b_{ij} = 0 \; \forall\, i \in \Gamma_N, \tag{1}$$

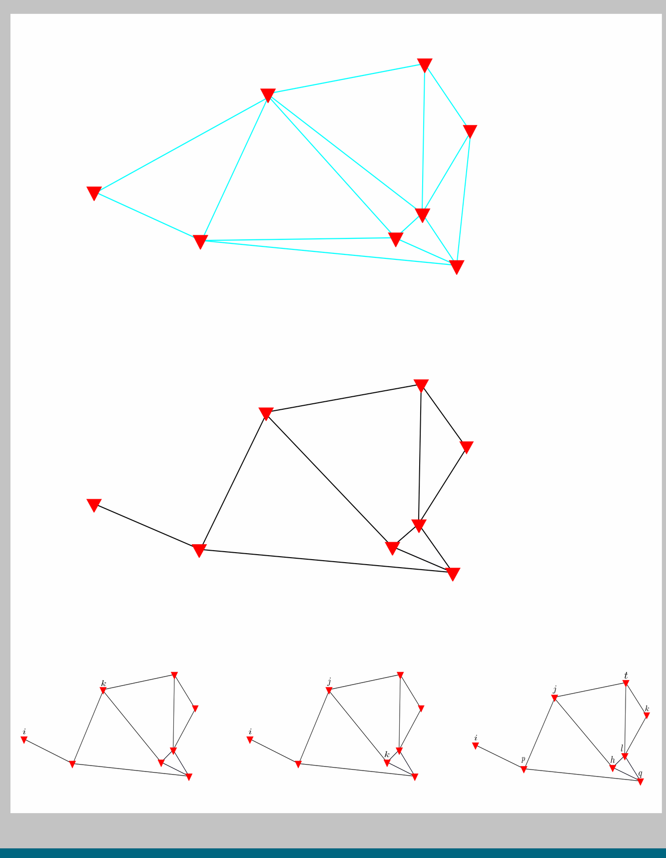
where, $\forall\, j \in \Delta_i$, holds:

$$P_{ij} = \left\{ \{p,q\} \middle| \; \{p,q\} \equiv e_{pq} \text{ and } p,q \in \{i\} \bigcup \Delta_i \; \right\}, \tag{2}$$

then:

$$\sum_{i \in \Gamma_N} \sum_{k \in \Gamma_N} b_{ik} = 0.$$

Lemma 1: If end-to-end connectivity is ensured over Delaunay neighbours via Delaunay edges, then end-to-end connectivity over the network is ensured.



Attempt to establish connectivity in a distributed manner via the local Delaunay edges (Feasible Paths)

$$\Pi_{ij} = \left\{ P_{ij} = \{i, \ldots, p, q, \ldots, j\} \middle| \begin{array}{l} p, q \in \{i\} \bigcup \Delta_i \\ \{p,q\} \equiv e_{pq} \end{array} \right\}.$$

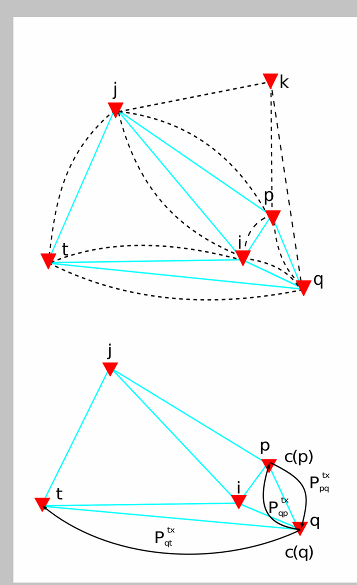
Problem 2: For each node $i \in \Gamma_N$ select the path $\pi_{ij} \in \Pi_{ij}$ that minimizes the total transmission power $\varrho_{ij}^{tx}\big|_{\pi_{ij}}$:

$$\pi_{ij} \in \Pi_{ij} : \min_{P_{ij} \in \Pi_{ij}} \left\{ \varrho_{ij}^{tx} \big|_{P_{ij}} \right\} = \varrho_{ij}^{tx} \big|_{\pi_{ij}} .$$

Shortest Path Algorithm at each node. (convergence in linear time)

Table 1: The Shortest Power Path Algorithm.

For $i \in \Gamma_N$, $j \in \Delta_i$:	
STEP 1.	$\forall q \in V_{ij} \setminus \{i\}$, set the label $c_l(q) = P_{iq}^{tx} + P_{qi}^{tx}$ and the origin node $o(q) = i$. Initialize the set of visited vertices $V_{ij}^{'} \subseteq V_{ij}$ and the optimal path set π_{ij} such that: $V_{ij}^{'} \leftarrow \{i\}$ and $\pi_{ij} \leftarrow \emptyset$.
STEP 2.	Select the vertex $p \in V_{ij} \setminus V_{ij}^{'}$ with the minimum label $c_l(p)$. If $p = j$, then construct the path π_{ij} according to: $\pi_{ij} = \{i, \ldots, o(o(j)), o(j), j\}$ and stop. Else go to STEP 3.
STEP 3.	For all vertices $q \in V_{ij} \setminus V_{ij}^{'}$, adjacent to the p-th vertex calculate the label $c_l(q)\|_p$ at the q-vertex, with respect to the p-th vertex: $c_l(q)\|_p = c(p) + P_{qp}^{tx}$. If $c_l(q)\|_p < c_l(q)$, then update the label $c_l(q)$ and the origin node $o(q)$ at the q-th vertex according to: $c_l(q) \leftarrow c_l(q)\|_p$ and $o(q) = p$. Return to STEP 2.



Each node calculates locally the optimal path and transmission power for establishing end-to-end connectivity with its Delaunay neighbours
The final value of the transmission power for each node is the one that guarantees connectivity with all Delaunay neighbours over Delaunay edges

$$\varrho_i^{tx} = \max_{j \in \Delta_i} \left\{ \varrho_{ij}^{tx} \right\}. \; (3)$$

Distributed Delaunay Connectivity Algorithm

$\forall i \in \Gamma_N$ and given the Delaunay tessellation $(\Delta_i, \mathcal{T}_i)$:	
STEP 1.	$\forall j \in \Delta_i$ calculate the feasible paths set Π_{ij}. Derive the graph $G_{ij} = (V_{ij}, E_{ij})$. Following the procedure described in Table 1 calculate the pair $(\pi_{ij}, \varrho_{ij}^{tx}\|_\pi$
STEP 2.	Select the value of ϱ_i^{tx} (Eq. (3)).
STEP 3.	If $\varrho_i^{tx} > \varrho^{max}$, then set $\varrho_i^{tx} \leftarrow \varrho^{max}$.

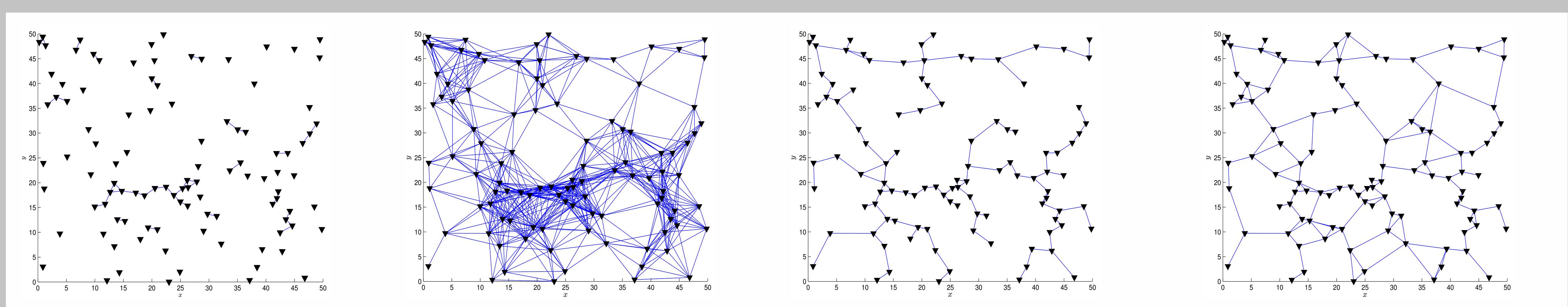
(convergence in quadratic time)

Simulation Studies

Simulation Snapshot G_N, N =104

Simulation set:
Network size varying from 4 to 299 nodes
Applied to 10 different sets of spatial distribution

Proposed Scheme compared against:
a. operation at maximum transmission power
b. MST resulting from the operation at maximum transmission power (Centralized-MST, CMST)



(A) Operation at minimum transimmion power:
End-to-end connectivity is not achieved.

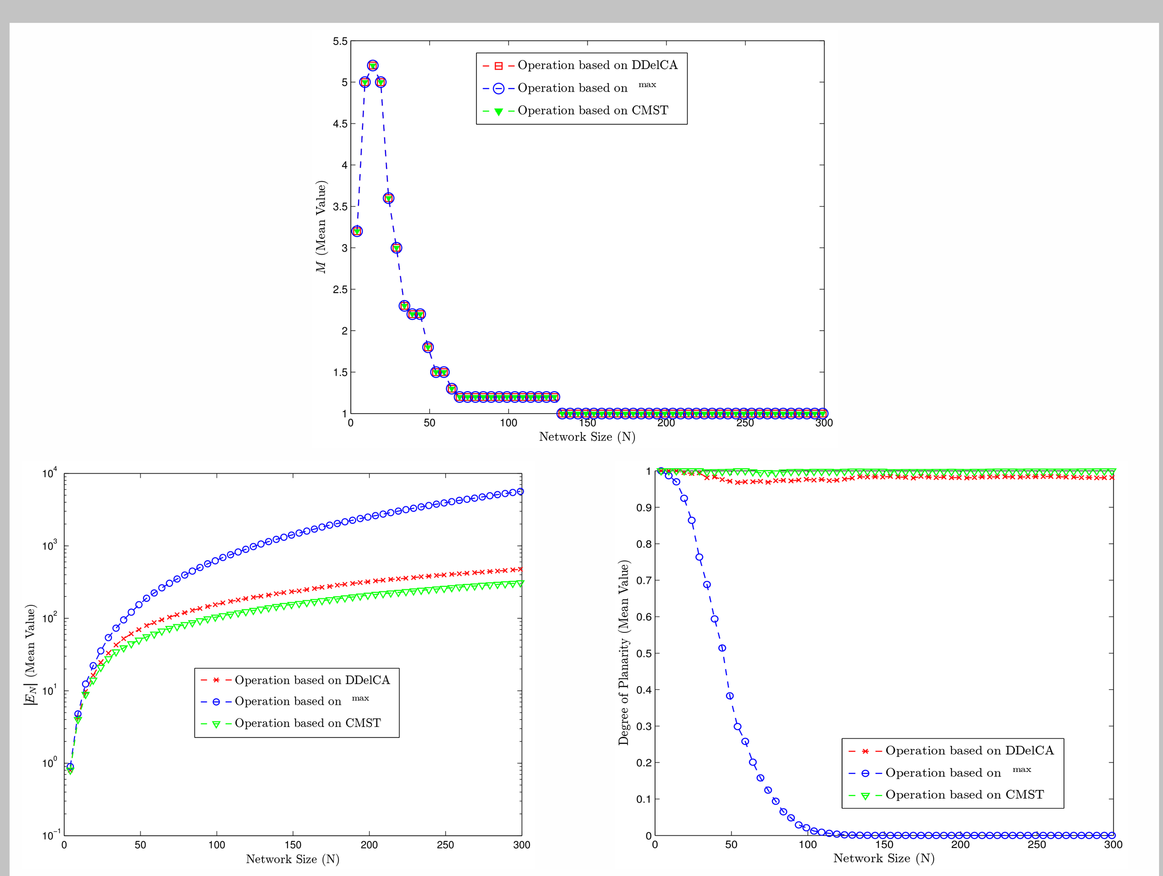
(B) Operation at maximum transimmion power:
End-to-end connectivity is achieved, with the cost of extraordinary edges.

(C) Operation based on MST:
End-to-end connectivity is achieved
Optimal solution, based on centralized knowledge.

(D) Operation based as Distributed Delaunay Connectivity:
End-to-end connectivity is achieved.
Sub-optimal (w.r.t. MST).
Derived based on local Delaunay awareness.
Redundant edges are reduced.

Simulation Results

End-to-End Connectivity



End-to-End Connectivity
The proposed scheme offers same end-to-end connectivity as the one provided by the operation at the maximum transmission power, with less redundant edges.
As the network size increases the proposed scheme converges to a planar graph (as the MST is).

Power Consumption and Lifetime
Significant improvement in both power consumption and lifetime, w.r.t. to the network's operation in maximum transmission power. Despite its suboptimality, the proposed scheme follows the behavior of the MST (optimal solution), and both power consumption and lifetime converge to the same level as the network size increases. 100% increase to transmission and reception rate does not significantly affect the power consumption and lifetime during the operation of the network based on the proposed scheme.

Power Consumption & Lifetime

$f_{TX} = f_{RX}$ =500bps $\qquad$ $f_{TX} = f_{RX}$ =1000bps