# High gain observer design for some networked control systems

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## Abstract

New results on emulation of a high gain observer over networks are presented. Using a general framework and a suitable Lyapunov function, some explicit conditions on the maximum allowable transmission interval are given for a large class of network protocols.

## Observers for networked control systems

Consider the following systems

$$\begin{cases} \dot{x} = f_p(x) \\ y = h_p(x) \end{cases} \tag{1}$$

and suppose that there exists an exponential observer

$$\dot{\hat{x}} = f_o(\hat{x}, (h_p(\hat{x}) - y)) \tag{2}$$

Then, the implementation of the above observer over a network can be done according to the following framework

$$\begin{cases} \dot{x} = f_p(x) \quad t \in [t_{s_{i-1}} \quad t_{s_i}] \\ x(t_{s_i}^+) = x(t_{s_i}) \\ y = h_p(x) \\ \dot{\hat{x}} = f_o(\hat{x}, \hat{w} - w) \quad t \in [t_{s_{i-1}} \quad t_{s_i}] \\ \hat{x}(t_{s_i}^+) = \hat{x}(t_{s_i}) \\ \dot{w} = g_w(w, \hat{w}, \hat{x}) \quad t \in [t_{s_{i-1}} \quad t_{s_i}] \\ w(t_{s_i}^+) = y_s(t_{s_i}) + h_y(i, e_y(t_{s_i})) \\ \dot{\hat{w}} = \hat{g}_{\hat{w}}(\hat{w}, w, \hat{x}) \quad t \in [t_{s_{i-1}} \quad t_{s_i}] \\ \hat{w}(t_{s_i}^+) = h_p(\hat{x}(t_{s_i})) \\ \dot{y}_s = g_{y_s}(y_s, x) \quad t \in [t_{s_{i-1}} \quad t_{s_i}] \\ y_s(t_{s_i}^+) = y(t_{s_i}) \\ e_y = w - y_s \end{cases} \tag{3}$$

The monotonically increasing sequence $t_{s_i}, i \in \mathbf{N}$ represents the transmission instants, we define the strictly positive variable $\tau = \max(t_{s_{i+1}} - t_{s_i})$ as the maximum allowable transfer interval (MATI). The vectors $w$ and $\hat{w}$ represent respectively most recently transmitted output values via network and the observer output; The vector $y_s$ is used in order to describe the sampling phenomena without network. In this case we replace conjointly the outputs of the system and the observer by their sampled signals before considering the constraints introduced by the network. This method allows us to derive, for several classes of nonlinear systems, an asymptotic stability of the observation error when the network operates in zero order hold (ZOH) fashion. The functions $g_{y_s}$, $g_w$ and $\hat{g}_{\hat{w}}$ represent the prediction functions of the network between two transmission instants. The error induced by the network is represented by the vector $e_y$. The protocol $h_y(i, e_y)$ is the algorithm by which the access to network of each nodes of actuators or sensors is determined. At each $t_{s_i}$, the protocol $h_y$ selects which nodes $j \in \{1, \ldots, l\}$ can transmit his data throughout the network.

## Triangular multi-output systems

$$\begin{cases} \dot{x} = Ax + f(x) \\ y = Cx \end{cases} \tag{4}$$

$$A = \begin{pmatrix} 0_p & I_p & 0_p & \ldots & 0_p \\ 0_p & 0_p & I_p & 0_p & \vdots \\ \vdots & 0_p & \ldots & I_p & 0_p \\ 0_p & \vdots & \ldots & 0_p & I_p \\ 0_p & \ldots & \ldots & \ldots & 0_p \end{pmatrix} \tag{5}$$

and

$$C = \begin{pmatrix} I_p & 0_p & \ldots & 0_p \end{pmatrix} \tag{6}$$

we assume that the following hypotheses are satisfied:

- The functions $f_i(x)$ are Lipshitz and have the following form

$$f(x) = \begin{pmatrix} f_1(x^1) \\ \vdots \\ f_k(x^1, \ldots, x^i) \\ \vdots \\ f_q(x) \end{pmatrix} \tag{7}$$

## High gain observer with an output predictor

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + f(\hat{x}) - \theta\Delta^{-1}S^{-1}C^T(\hat{w} - w) \\ \dot{w} = \hat{x}^2 + f_1(\hat{x}^1) \quad t \in [t_{s_{i-1}} \quad t_{s_i}] \\ w(t_{s_i}^+) = y_s(t_{s_i}) + h_y(i, e_y(t_{s_i})) \\ y_s = y \\ \hat{w} = C\hat{x}(t) \end{cases} \tag{8}$$

## High gain observer in ZOH fashion

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + f(\hat{x}) - \theta\Delta^{-1}S^{-1}C^T(\hat{w} - w) \\ \dot{w} = 0 \quad t \in [t_{s_{i-1}} \quad t_{s_i}] \\ w(t_{s_i}^+) = y_s(t_{s_i}) + h_y(i, e_y(t_{s_i})) \\ \dot{\hat{w}} = 0 \quad t \in [t_{s_{i-1}} \quad t_{s_i}] \\ \hat{w}(t_{s_i}^+) = C\hat{x}(t_{s_i}) \\ \dot{y}_s = 0 \quad t \in [t_{s_{i-1}} \quad t_{s_i}] \\ y_s(t_{s_i}^+) = y(t_{s_i}) \\ e_y = w - y_s \end{cases} \tag{9}$$

## Exponential convergence

we assume that the following hypothesis is satisfied:

- $h_y$ is an UGES protocol

Then, there exist bounds of MATI such $\forall \tau < \tau_{MATI}$ the above observers converge exponentially towards the systems.

**This result is derived using a Lyapunov approach adapted to impulsive systems.**