# Distributed Change Detection Based on a Consensus Algorithm

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## Motivation

Change detection is one of typical tasks of sensor networks; possibility to test the decision variables **at any node in the network** and **in real time** is often desirable.

• In the classical multi-sensor detection schemes the local sensors send all their data to other sensors, and ultimately to a fusion center - the decision variables are tested only at predefined fusion nodes.

• Most of the recent attempts to apply consensus techniques to the distributed detection problem assume that the dynamic agreement process starts after all data have been collected - inapplicable to real time change detection problems.

## Contribution

A novel algorithm is proposed for *distributed change detection* while monitoring the environment through a wireless sensor network.

• All the nodes in the network generate local decision variables by recursive schemes belonging to the *geometric moving average control charts*, applicable in **real time**.

• A dynamic consensus scheme with preselected asymmetric communication gains is applied; an algorithm which asymptotically provides nearly equal behavior of all the nodes is obtained (*i.e.*, **any node** can be selected for testing the decision variable w.r.t. a pre-specified threshold).

## Algorithm

• Network with *n* nodes, each node collects measurements and generates at each discrete time instant *t* a scalar quantity $x_i(t)$, directly or as a result of local signal processing; $\{x_i(t)\}$ are considered as mutually independent stationary random sequences with means $m_i$ and covariances $r_i(\tau)$.

• Global decision function for the whole network

$$s_c(t+1) = \alpha\, s_c(t) + (1-\alpha)\sum_{i=1}^{n}\omega_i x_i(t+1), \quad s_c(0)=0, \quad 0<\alpha<1,$$

where $\omega_i = k\theta_i^1\sigma_i^{-2}$ are the components of the vector $\omega^T = k\theta^{1T}\Sigma^{-1}$ ($k = (\sum_{i=1}^n w_i)^{-1}$).

•The basic assumption: nodes in the network are connected in such a way that the $n\times n$ matrix *C* represents the weighted adjacency matrix for the underlying graph representing the network, and that *C* is *row stochastic.*

• The proposed algorithm generates the vector decision function $s(t) = [s_1(t)\cdots s_n(t)]^T$ of the network:

$$s(t+1) = \alpha\, Cs(t) + (1-\alpha)Cx(t+1), \quad s(0)=0, \qquad \text{where } x(t) = [x_1(t)\cdots x_n(t)]^T.$$

• Consensus matrix *C* performs for each node "convexification" of the neighboring states and enforces in such a way consensus between the nodes.

## Convergence analysis

• Assumptions:

A1) *C* has the eigenvalue 1 with multiplicity 1;

$\Rightarrow$ $C^i$ converges to a nonnegative row stochastic matrix with equal rows;

A2) $\lim_{i\to\infty} C^i = \mathbf{1}\omega^T$;

$\Rightarrow$ *C* can be constructed by solving the linear equation $\omega^T C = \omega^T$ under the constraints that some of the elements of *C* are equal to zero and that it is row stochastic.

• Error is defined as $e(t) = s(t) - \mathbf{1}s_c(t) = (I-\mathbf{1}\omega^T)s(t) = (1-\alpha)\sum_{i=0}^{t-1}\alpha^i\widetilde{C}^{i+1}\widetilde{x}(t-i)$,
where $\widetilde{C} = C - \mathbf{1}\omega^T$, $\widetilde{x}(t) = (I-\mathbf{1}\omega^T)x(t)$.

• $s(t)$ as the estimator of $s_c(t)$ is, in general, biased; $m_e = E\{e(t)\} = 0$ only when $m_i = m_j,\ i,j=1,\dots,n$; the bias is smaller when $\alpha$ is closer to 1 ($E\{e(t)\} \sim (1-\alpha)$).

• The focus of the analysis is placed on the mean-square error matrix,
defined as $Q(t) = E\{e(t)e(t)^T\} - m_e(t)m_e(t)^T = (1-\alpha)^2\Phi(t)^T\widetilde{R}(t)\Phi(t)$, where
$\Phi(t) = [\alpha^{t-1}\widetilde{C}^t \vdots \alpha^{t-2}\widetilde{C}^{t-1} \vdots\cdots\vdots \alpha^0\widetilde{C}]^T$, $\widetilde{R}(t) = R(t) - m_X m_X^T$, $R(t) = E\{X(t)X(t)^T\}$,
$X(t) = [x(1)^T\cdots x(t)^T]^T$, $m_X = E\{X(t)\}$; further, $\widetilde{R}(t) = [R_{ij}]$, where
$R_{ij} = \mathrm{diag}\{r_1(i-j),\dots,r_n(i-j)\}$.

*Theorem 1.*
Let assumptions A1) and A2) hold, together with A3) $\max_i \sum_{\tau=0}^{\infty}|r_i(\tau)| < K < \infty$.
Then,

$$\max_{i,j} Q_{ij}(t) = O((1-\alpha)^2).$$

*Proof:*

• The n-dimensional quadratic form $y^TQ(t)y = (1-\alpha)^2y^T\Phi(t)^T\widetilde{R}(t)\Phi(t)y$ is analyzed.

• The expression $y^T\Phi(t)^T\Phi(t)y$ is in the form of a sum of terms containing $y^T\widetilde{C}^i\widetilde{C}^{iT}y$;

• From assumptions A1) and A2) $\Rightarrow \widetilde{C}$ has the same eigenvalues as C, except for the eigenvalue 1 which is replaced by 0 $\Rightarrow$ modules of all of its eigenvalues are less than 1;

• The recursion $P(t+1) = \widetilde{C}P(t)\widetilde{C}^T$, $P(0) = I$ is considered ( $P(t) = \widetilde{C}^t\widetilde{C}^{tT}$);

• The column vectors of $P(t)$ are concatenated to obtain an $n^2$-vector $\mathrm{vec}\{P(t)\}$
$\Rightarrow \mathrm{vec}\{P(t+1)\} = (\widetilde{C}\otimes\widetilde{C})\mathrm{vec}\{P(t+1)\}$ ("$\otimes$" denotes the Kronecker's product);

• From A1) and A2) $\Rightarrow |\lambda(\widetilde{C}\otimes\widetilde{C})|_{\max} = \lambda_M < 1$ (eigenvalues of $\widetilde{C}\otimes\widetilde{C}$ take values from the cross products of the eigenvalues of $\widetilde{C}$ ) $\Rightarrow \|P(t)\| \le k_p\lambda_M^t$
$\Rightarrow y^T\widetilde{C}^i\widetilde{C}^{iT}y \le k_p\lambda_M^i\|y\|^2$ and $y^T\Phi(t)^T\widetilde{R}(t)\Phi(t)y \le \|y\|^2k'K\sum_{i=0}^{\infty}\alpha^i\lambda_M^{i+1} \le K_1 < \infty$

• By choosing $y = e_i$ ( $e_i$ denotes the *n*-vector of zeros with only i-th entry equal to one)
$\Rightarrow Q_{ii}(t) \le K_1(1-\alpha)^2$; furthermore, $|Q_{ij}(t)| \le \max_i Q_{ii}(t)$. Q.E.D.

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## Distributed detection based on time averaging

• The recursive algorithms with constant coefficient $\alpha$ are essentially tracking algorithms with exponential forgetting, able to cope with change detection phenomena.

• In the case when $\alpha$ is a function of time tending to 1 when $t$ tends to infinity, the algorithms are not directly suitable for change detection purposes.

*Theorem 2.* Let $\alpha$ be replaced by $\alpha(t+1) = 1-\gamma(t+1)$ , and let the assumptions A1), A2) and A3) be satisfied, together with:
A4) $\gamma(t)$ is a non-increasing sequence satisfying
$\gamma(t) > 0,\ \lim_{t\to\infty}\gamma(t) = 0,\ \sum_{t=1}^{\infty}\gamma(t) = \infty$.
Then,
$\|Q(t)\| = o(1)$.

*Theorem 3.* Under the assumptions of Theorem 2 and with $\gamma(t) = \frac{1}{t}$ we have
$\|Q(t)\| = o(t^{-2})$
while $\mathrm{var}\{s_c(t)\} = O(t^{-1})$.

## Simulation results

• Network with *n* = 10 nodes is considered, where the means are randomly taken from the interval [0, 1], and variances randomly taken from the interval [0.5, 1.5] (means are zero in the case of no change). The moment of change is chosen to be *t* = 200.

• Communication gains are obtained by solving the linear equation $\omega^T C = \omega^T$.

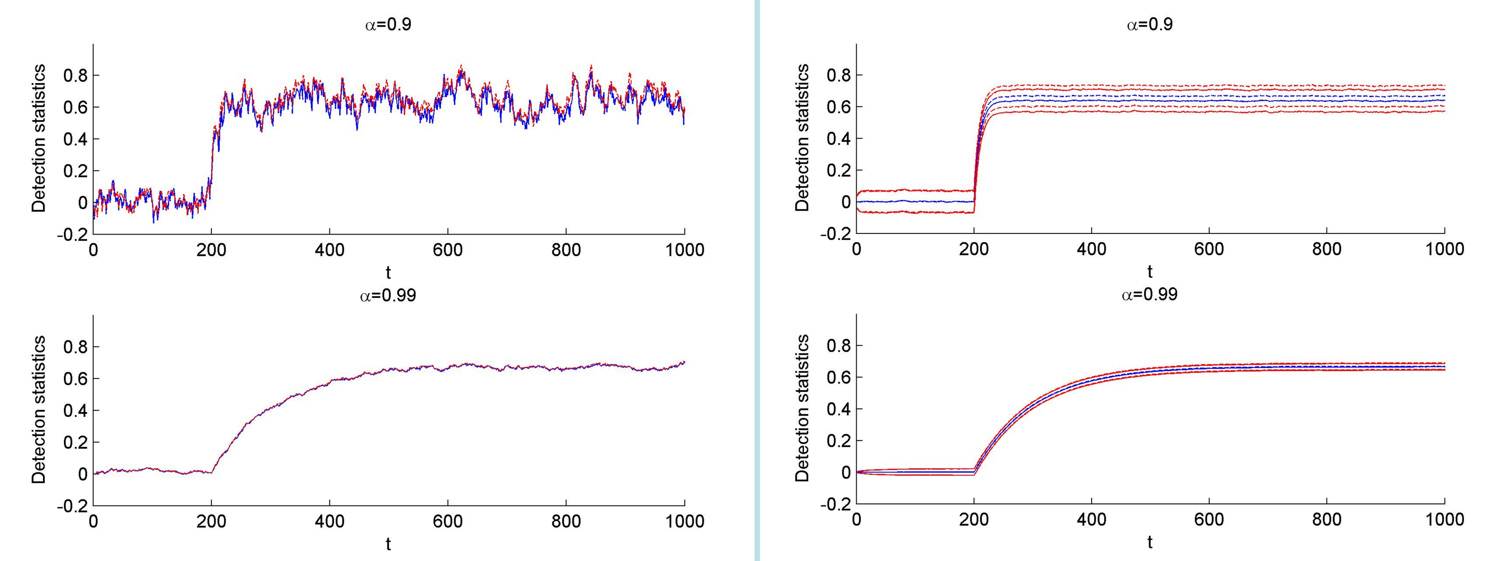
• *e.g.*

$$\omega_i = \frac{\theta_i^1\sigma_i^{-2}}{\sum_{i=1}^n\theta_i^1\sigma_i^{-2}} \Rightarrow C =$$

( $\lim_{i\to\infty} C^i = \mathbf{1}\omega^T$ )

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0.4500 | 0 | 0 | 0.1291 | 0 | 0 | 0.1314 | 0.1683 | 0 | 0.1212 |
| 0 | 0.2761 | 0.2930 | 0 | 0 | 0 | 0.0942 | 0.2955 | 0 | 0.0411 |
| 0 | 0.2770 | 0.2944 | 0 | 0 | 0 | 0.0909 | 0.2997 | 0 | 0.0380 |
| 0.0176 | 0 | 0 | 0.1352 | 0 | 0.1609 | 0 | 0.2664 | 0.3233 | 0.0966 |
| 0 | 0 | 0 | 0 | 0.2226 | 0.2569 | 0 | 0 | 0.5205 | 0 |
| 0 | 0 | 0 | 0.1003 | 0.1076 | 0.1247 | 0.1129 | 0.2396 | 0.3149 | 0 |
| 0.0109 | 0.2191 | 0.2265 | 0 | 0 | 0.1213 | 0.1195 | 0.2287 | 0 | 0.0741 |
| 0.0013 | 0.2396 | 0.2570 | 0.0572 | 0 | 0.0869 | 0.0651 | 0.2689 | 0 | 0.0240 |
| 0 | 0 | 0 | 0.0903 | 0.1080 | 0.1371 | 0 | 0 | 0.6646 | 0 |
| 0.0253 | 0.1942 | 0.1976 | 0.1341 | 0 | 0 | 0.1427 | 0.2000 | 0 | 0.1062 |

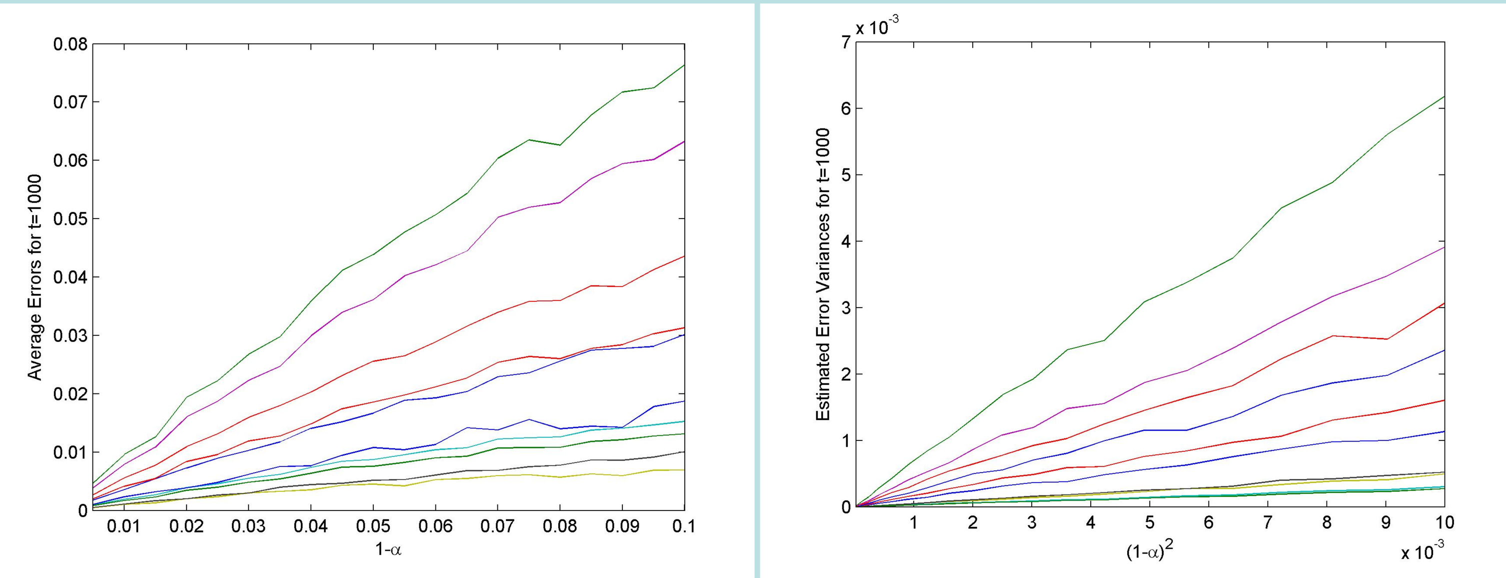
• The proposed algorithm effectively achieves very similar behavior of all of the nodes, with local decision functions getting closer to the global decision function as $\alpha \to 1$.




Decision functions – one realization: for one node (blue) and global (red)

Decision functions – mean +/- one standard deviation: for one node (solid lines) and global (dashed lines)

•The values of $|E\{e(t)\}|$ and $E\{e(t)e(t)^T\}_{ii}$ are estimated for different values of $\alpha$ for *t* = 1000 using 1000 Monte Carlo runs.

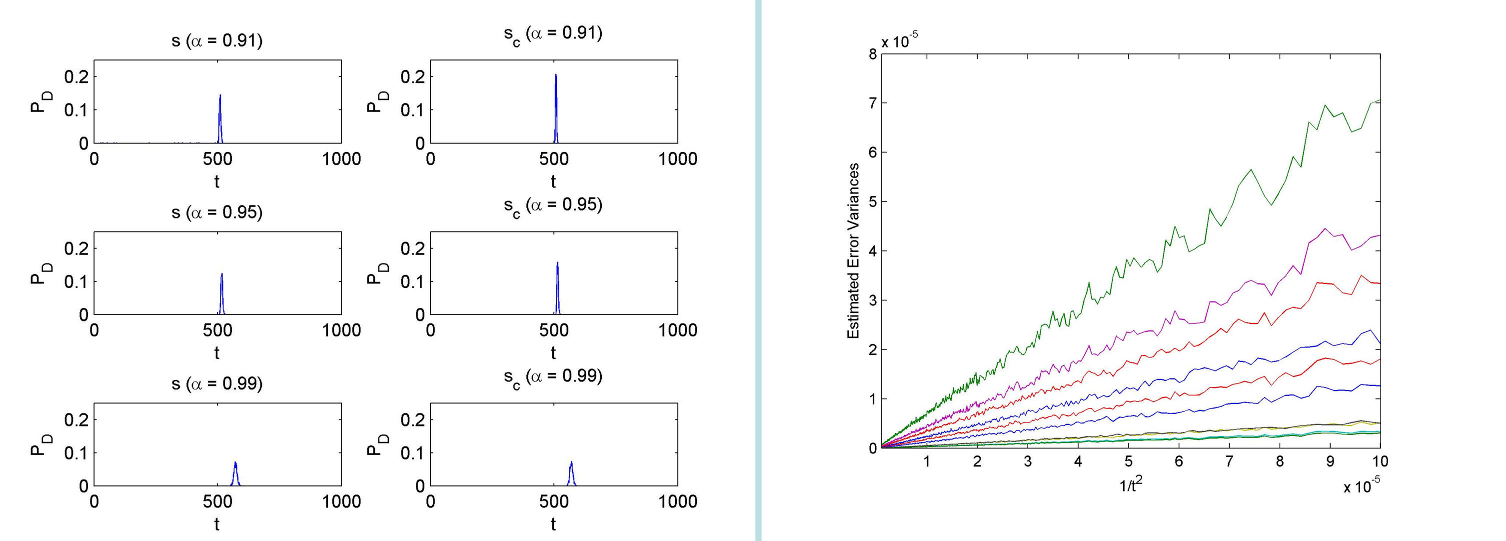



Average errors as functions of $\alpha$

Estimated error variances as functions of $\alpha$

• The distribution of the alarm times at which a detection occurs is estimated for different values of $\alpha$ (the moment of change is *t* = 500, the threshold is $\lambda_c = 0.5k\theta^{1T}\Sigma^{-1}\theta^1$).

• In the time averaging case the estimates of $E\{e(t)e(t)^T\}_{ii}$ are calculated using 1000 Monte Carlo runs.




The distribution of the alarm times for different values of $\alpha$: for one node (left) and global (right).

Estimated error variances as functions of time

**FURTHER WORK**: Generalization of the presented results to the case of stochastic time varying consensus matrices and application of the same methodology to the recursive Generalized Likelihood Ratio (GLR) algorithm.