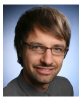
Suboptimal Event-based Control of Linear Systems over Lossy Channels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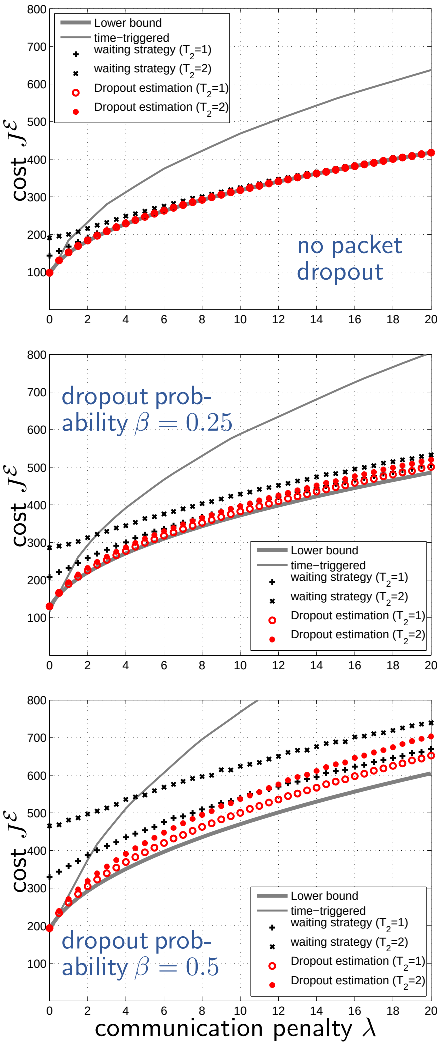
Motivation

Networked control systems pose the need for a **conjoint** consideration of

- Control performance
- Constrained communication
- Packet loss and time-delay

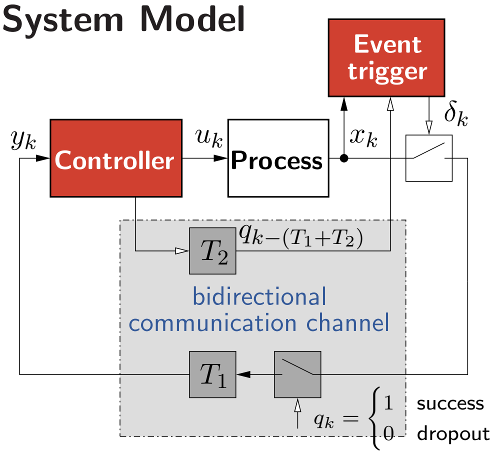
Numerical validation

Scenario: $A = B = 1,\ x_0, w_k \sim \mathcal{N}(0,1)$
$T_1 = 1,\ Q = Q_N = 1,\ R = 10,\ N = 100.$



Observation: DE outperforms WS for low λ.

System Model



- Stochastic discrete-time process:
  $x_{k+1} = Ax_k + Bu_k + w_k$
  $y_{k+T_1} = \begin{cases} x_k, & \delta_k = 1 \wedge q_k = 1 \\ \emptyset, & \text{otherwise} \end{cases}$
- Time-invariant controller
  $u_k = \gamma_k(\mathcal{I}_k^{\mathcal{C}})$
- Event-trigger
  $\delta_k = \mathrm{f}_k(\mathcal{I}_k^{\mathcal{E}}) = \begin{cases} 1 & \text{send } x_k \\ 0 & \text{idle} \end{cases}$
- $\mathcal{I}_k^{\mathcal{C}}$ and $\mathcal{I}_k^{\mathcal{E}}$ observation history

Objective

Design f, γ minimizing $J(\mathrm{f},\gamma) = \mathsf{E}\left[x_N^{\mathrm{T}} Q_N x_N + \sum_{k=0}^{N-1} x_k^{\mathrm{T}} Q x_k + u_k^{\mathrm{T}} R u_k + \overbrace{\lambda \delta_k}^{\text{comm. penalty}}\right]$

Suboptimal approaches

Waiting Strategy (WS)

Restriction: Allow only **1** unacknowledged packet (congestion window=1).

Define additional discrete state to flag an unacknowledged packet

$$s_{k+1} = \begin{cases} T_1+T_2-1 & \delta_k=1 \wedge s_k=0 \\ s_k-1 & \delta_k=0 \wedge s_k>0 \\ 0 & \delta_k=0 \wedge s_k=0 \end{cases}$$

Optimal solution:

$$u_k = \gamma_k^*(\mathcal{I}_k^{\mathcal{C}}) = -L_k\,\mathsf{E}[x_k|\mathcal{I}_k^{\mathcal{C}}] \qquad (1)$$

Gain L_k is related to LQ regulation.

$$\min_f \mathsf{E}\left[\sum_{k=0}^{N-1} e_k^{\mathrm{T}}\Gamma_k e_k + \lambda\delta_k\right], \qquad (2)$$

$$e_{k+1} = (1 - \mathbb{1}_{\{s_k=0\}} q_k \delta_k) A e_k + w_k$$

Estimation error $e_k = x_k - \mathsf{E}[x_k|\mathcal{I}_k^{\mathcal{C}}]$ is **sufficient statistic** for event-trigger f.

Key: Dividing initial problem into tractable subproblems.

Dropout Estimation (DE)

Restriction: Assume certainty equivalence controller given by (1).

Problem reduction:
Based on [Matei et al. 2008] the arguments of the optimal event-trigger reduce to

(i) finite history of state and events $X^k_{k-\max(T_1,T_2)}, \delta^{k-1}_{k-\max(T_1+T_2)}$

(ii) last known estimate $\mathsf{E}[x_{k-T_2}|\mathcal{I}^{\mathcal{C}}_{k-T_2}]$

Main Insights

- Initial problem numerically infeasible
- Within subclass WS and DE: Optimal strategy computable
- DE: Comp. complexity increases exponentially with **round-trip time**
  $\Delta_{\text{RRT}} = T_1 + T_2$
- WS: Scale-free complexity w.r.t. Δ_{RRT}
- WS and DE: **Technological realization** as extension of TCP
- **Further analysis:** Network stability

References:

- A. Molin, S. Hirche, "On LQG Joint Optimal Scheduling and Control under Communication Constraints", *Proceedings of the 48th IEEE International Conference on Decision and Control*, 2009.
- I. Matei, N. Martins, and J. Baras, "Optimal State Estimation for Discrete-Time Markovian Jump Linear Systems in the Presence of Delayed Output Observations", *American Control Conference 2008*, 2008.

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