Distributed collision avoidance for interacting vehicles: a command governor approach

Francesco Tedesco[1], Davide M. Raimondo[2], Alessandro Casavola[3], John Lygeros[4]

[1] ftedesco@deis.unical.it, [2]davide.raimondo@control.ee.ethz.ch, [3] casavola@deis.unical.it, [4]lygeros@control.ee.ethz.ch

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Abstract

This paper deals with a distributed coordination problem including collision avoidance. The problem is solved by using a Command Governor strategy based on mixed integer optimization. First, we present an algorithm to find an appropriate command in the centralized case, then a distributed sequential procedure is described. Simulations are reported for comparisons.

1. Problem Formulation

SYSTEM DESCRIPTION:

Consider a set of *N* decoupled sytems $\mathcal{A}=\{1,\dots,N\}$ where i-th system is described by the following equations:

$$\begin{cases} x_i(t+1) &= \Phi_{ii}x_i(t)+G_i g_i(t) \\ z_i(t) &= H_i^z x_i(t) \\ c_i(t) &= H_i^c x_i(t)+L_i g_i(t) \end{cases}$$

- x_i *is the state vector*
- $z_i(t) := [z_i^x(t), z_i^y(t)]^T$ *is the output vector that is required to track the reference vector* r_i *and it's subject to collision avoidance constraint*

$$||z_i(t)-z_j(t)||_\infty > d \quad \forall i,j(i\neq j) \quad and \quad t\in\mathbb{Z}_+$$

- $g_i(t) := [g_i^x(t), g_i^y(t)]^T$ *is the manipulable reference vector that, in the absence of constraints, equals* r_i
- c_i *is the local constrained vector that* ***has to fulfill the set membership constraints*** $c_i \in \mathcal{C}_i$ *where* $\mathcal{C}_i$ *is a convex and compact set.*

ASSUMPTIONS:

A1. Each system is asymptotically stable (i.e. the system is pre-compensated)
A2. Each system is offset free

AGENTS AND COMMUNICATION NETWORK:

- Each subsystem is governed by an independent agent.
- Agents are connected by means of a communication network modeled by an undirected communication graph $\mathcal{G}=(\mathcal{A},\mathcal{B})$.
- It is assumed that two connected agents can directly share information.

GOAL: Locally determine, at each time *t* and for each agent, a modified reference signal g_i as the best approximation of the desired reference r_i that ensures constraints fulfilment.

2. Centralized Command Governor for collision avoidance

Standard centralized solutions to the problem, without considering collision avoidance constraints, have been achieved in [1], [2].

Here we present a Command Governor able to handle collision avoidance constraints. Let consider the global system

$$\begin{cases} x(t+1) &= \Phi x(t)+Gg(t) \\ z(t) &= H^z x(t) \\ c(t) &= H^c x(t)+Lg(t) \end{cases}$$

where $x=[x_1^T,\dots,x_N^T]^T, r=[r_1^T,\dots,r_N^T]^T, g=[g_1^T,\dots,g_N^T]^T, z=[z_1^T,\dots,z_N^T]^T, c=[c_1^T,\dots,c_N^T]^T$ are the aggregate vectors arising from the composition of the N subsystems. The problem is to select a suitable reference $g(t) := \underline{g}(r(t),x(t))$ according to following constraints

$$c(t)\in\mathcal{C}$$
$$z(t)\in\mathcal{Z}$$

where $\mathcal{C} := \{\mathcal{C}_1\times\dots\times\mathcal{C}_N\}$ and $\mathcal{Z} := \{z : ||z_i-z_j||_\infty > d \;\; \forall i,j\in\mathcal{A}(i\neq j)\}$.

The set $\mathcal{Z}$ can be manipulated in order to obtain **mixed integer** /linear constraints. Hence z belongs to $\mathcal{Z}$ if it is contained in the *z*-projection of the set

$$\bar{\mathcal{Z}}(d) := \left\{ (z,T)\in\mathbb{R}^2\times\{0,1\}^4 \;\middle|\; \begin{array}{l} \forall i,j|i>j: \quad z_i^x - z_j^x \geq d-\mu T_{ij}^1 \\ and \; z_i^y - z_j^y \geq d-\mu T_{ij}^2 \\ and \; z_j^x - z_i^x \geq d-\mu T_{ij}^3 \\ and \; z_j^y - z_i^y \geq d-\mu T_{ij}^4 \\ and \quad \sum_{p=1}^{4} T_{ij}^p \leq 3 \end{array} \right\}$$

where $T := [T_{ij}^1, T_{ij}^2, T_{ij}^3, T_{ij}^4]$

The reference $g(t)$ is applied according a receding horizon fashion once chosen in a family of constant virtual sequences $g(\cdot)=\{g(k)\equiv g\in\mathcal{W}_\delta,\; \forall k\leq k_0\}$ in order that future predictions

$x(k,x(t),g) := \Phi^k x(t) + \sum_{i=0}^{k-1}\Phi^{k-i-1}Gg$, $c(k,x(t),g) := H^c x(k,x(t),g)+Lg$ and

$z(k,x(t),g) := H^z x(k,x(t),g)$ satisfy constraints within a fixed horizon k_0 and the equilibrium solutions $x_g := (I_n-\Phi)^{-1}Gg$ $\quad z_g := H^z(I_n-\Phi)^{-1}Gg$ $\quad c_g := H^c(I_n-\Phi)^{-1}Gg+Lg$ belong to the sets

$$\mathcal{C}^{\delta_1} := \mathcal{C}\sim\mathcal{B}_{\delta_1}$$
$$\bar{\mathcal{Z}}^{\delta_2} := \bar{\mathcal{Z}}(d+\delta_2)$$

Finally the reference *g* is a solution of the following optimization problem

$$\hat{g}(t) = \arg\min_{(g,T(\cdot))\in\mathcal{V}(x(t))} ||g-r(t)||^2_{\Psi_g}$$

where

$$\mathcal{V}(x(t)) = \{(g,T(\cdot))\in\mathcal{W}_\delta : (z(k,x(t),g),T(k))\in\bar{\mathcal{Z}}, \; c(k,x(t),g)\in\mathcal{C}, \forall k\leq k_0\}$$

and

$$\mathcal{W}_\delta := \left\{g\in\mathcal{R}^m : c_g\in\mathcal{C}^{\delta_1}, (z_g,T)\in\bar{\mathcal{Z}}^{\delta_2}_M, T\in\{0,1\}^4\right\}$$

3. Distributed CG for collision avoidance

Let the communication graph be an Hamiltonian graph and the cycle $H=\{1,2,\dots,N\}$ an Hamiltonian cycle.

The idea behind the S-CG is **to allow only one agent per time** to manipulate its local command signal. After each decision, the agent in charge transmits its local command and state to the next updating agent. Such a polling policy implies that, eventually after a preliminary initialization cycle, at each time instant the "agent in charge" always knows the whole aggregate vector $g(t-1)$ and the past whole aggregate state vector $x(t-1)$

S- CG Algorithm – AGENT i

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REPEAT AT EACH TIME t=kτ, k=0,1,…
IF (k mod N) == i
1.1 RECEIVE g(t−1) AND x(t−1) FROM THE PREVIOUS AGENT IN H
1.2 SOLVE
      g_i(t) = arg min_{g_i} ||g_i − r_i(t)||²_{Ψ_i}
      subject to: {([g_1^T(t−τ),…g_i,…,g_N^T(t−τ)^T]^T, T(·)) ∈ V(x(t))
1.3 APPLY g_i(t)
1.4 SEND THE UPDATED g(t) TO THE NEXT AGENT
ELSE
2.1 APPLY g_i(t) = g_i(t−τ)
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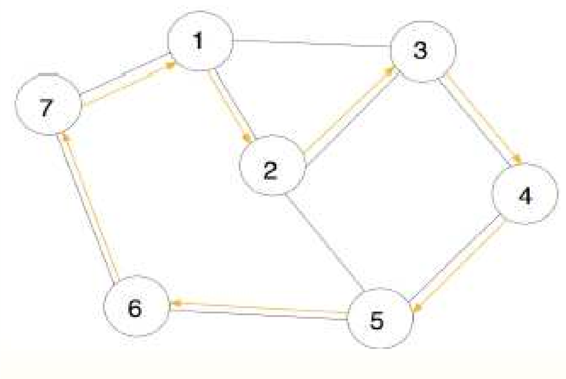


Graph $\mathcal{A}$ and Hamiltonian Cycle $\mathcal{H}$

4. Example

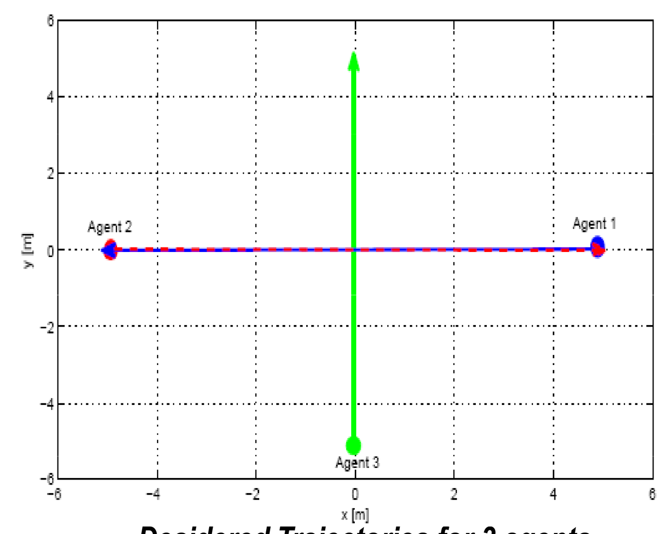
A system consisting of three decoupled particle masses is considered. The system is described by equations:

$$m_i\ddot{x}_i = F_i^x$$
$$m_i\ddot{y}_i = F_i^y$$

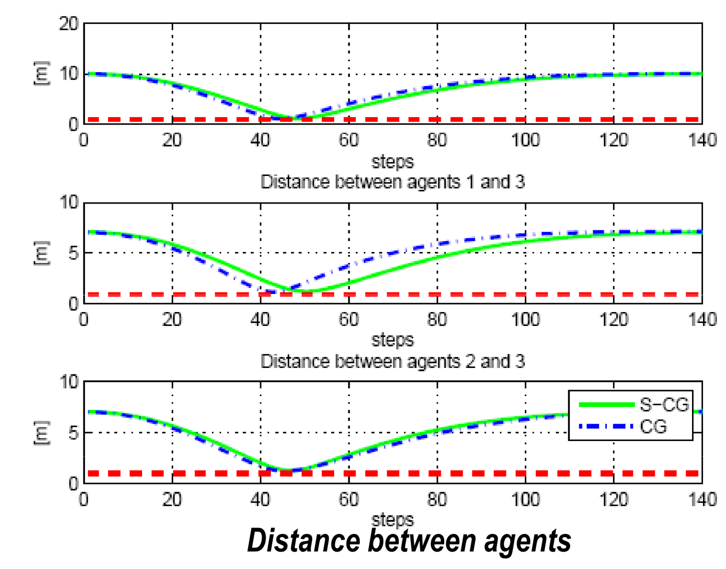
Where (x_i,y_i) $i\in\mathcal{A}=\{1,2,3\}$ are the coordinates of the i-th mass position w.r.t a cartesian reference and (F_i^x,F_i^y) the components along the same reference frame of the forces acting as inputs for subsystems.

These three masses are required to reach respectively points $r_1=[-5,0]^T$, $r_2=[5,0]^T$, $r_3=[0,5]^T$ complying with the following constraints:

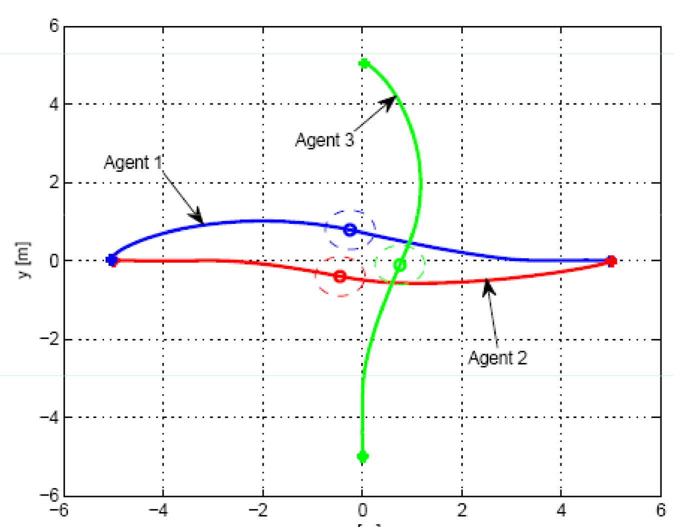
- Input saturation constraints: $|F_i^j(t)|\leq 2\;[N] \quad j=x,y,\; i\in\mathcal{A}$
- Collision Avoidance Constraints: $\max\{(x_i(t)-x_j(t)),(y_i(t)-y_j(t))\}\geq 1[m]\;\; i,j\in\mathcal{A}, i\neq j$



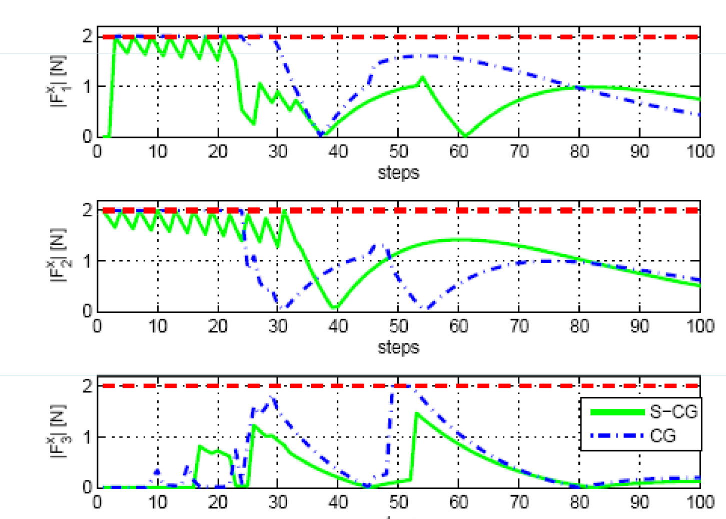
Desidered Trajectories for 3 agents



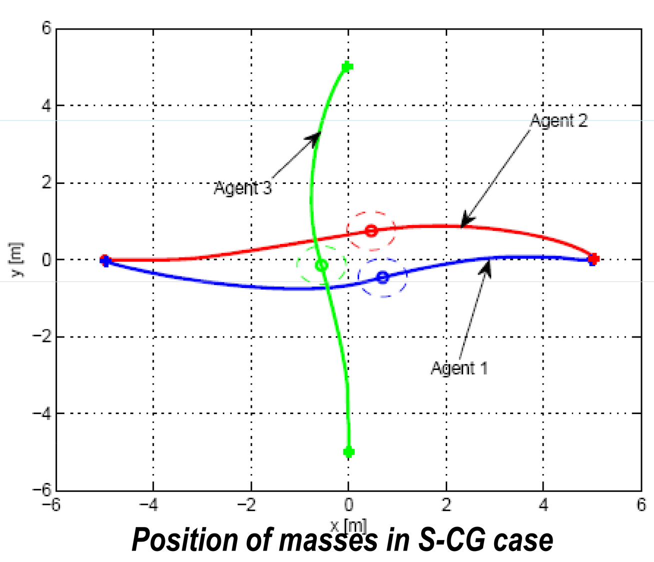
Distance between agents



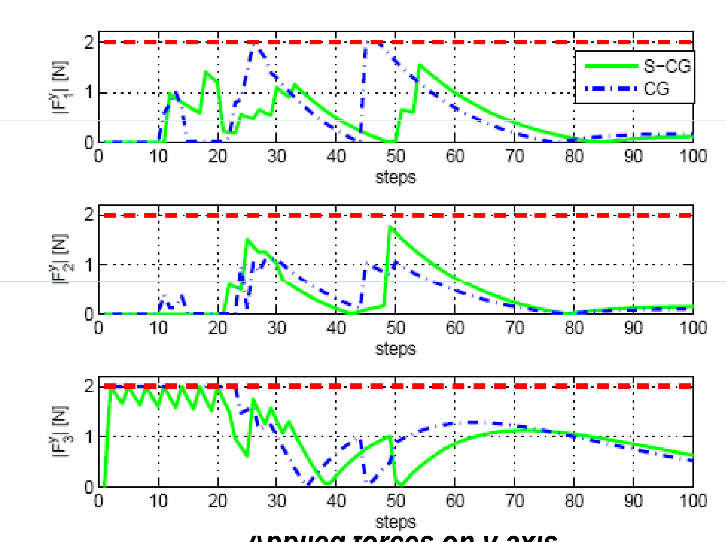
Position of masses in CG case



Applied forces on x axis



Position of masses in S-CG case



Applied forces on y axis

ROG scheme	CPU Time
CG	0.022
S-CG	0.0017

Numerical burdens: CPU time (seconds per step)

Acknowledgements

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References

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