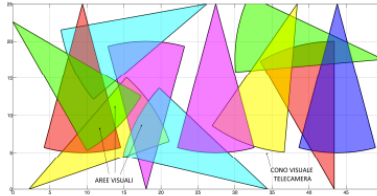


## The Graph Building Problem

### Distributed scenario:

- The knowledge of the topology of the network by the acting agents is of paramount importance
- Agents share information locally and need to coordinate with neighbors to attain global performance
- Unfeasible manual setup of medium/large scale networks with hundreds of nodes spread across the environment
- Necessity of learning the topology after the network has been deployed

**Camera networks:** how do the set of camera sensor scenes relate to the environment topology?



### Graph description:

- Each node represents a "sensor area"
- Each edge stands for an admissible physical transition from one area to another

### Aim:

- Estimate the graph structure in terms of both nodes (i.e. areas of interest) and edges (i.e. the transition probability map) from camera observations, during the system calibration phase

## Hidden Markov Model and Baum Welch Algorithm

### Assumptions:

- 2D domain and  $K$  fixed cameras  $\{A_i, i = 1, \dots, K\}$
- The observation is a binary string:  $O_t \in \{0, 1\}^K$  entry in the  $i$ -th position is 1 if the  $i$ -th camera sees the target object, while it is 0 otherwise
- Different observations imply that the corresponding states of the system are distinguishable

**Problem:** given an observation sequence  $O$  infer the set of states  $S$  and the underlying graph  $G$  that constrains their transitions (whose node set is  $S$ ).

### HMM:

- State set:  $S = \{S_1, S_2, \dots, S_N\}$
- Observation sequence:  $O = [O_1, \dots, O_T]$ ,  $O_i \in \{v_1, v_2, \dots, v_M\}$ ,  $v_i \in \{0, 1\}^K$
- State transition probability:  $A \in \mathbb{R}^{N \times N}$ ,  $a_{ij} = P[q_{t+1} = S_j | q_t = S_i]$
- Observation symbol probability:  $B \in \mathbb{R}^{M \times N}$ ,  $b_{ij} = P[O_t = v_i | q_t = S_j]$
- Initial state distribution:  $\pi_i = P[q_1 = S_i]$

→ Model:  $\lambda = (A, B, \pi)$

**Problem:** given an initial model  $\lambda_0$  adjust the model parameters  $\{A, B, \pi\}$  to maximize the probability of the observation sequence  $O$  given the model:

$$\max_{\lambda} P[O|\lambda]$$

### Baum-Welch algorithm:

- Expectation-maximization method
- Forward var.:  $\alpha_t(i) = P[O_1, O_2, \dots, O_t, q_t = S_i | \lambda]$
- Backward var.:  $\beta_t(i) = P[O_{t+1}, \dots, O_T | q_t = S_i, \lambda]$
- Estimation of the new model:  $\{\hat{A}, \hat{B}, \hat{\pi}\}$

$$P[O|\hat{\lambda}] \geq P[O|\lambda]$$

## Splitting and HMM Identification

### Procedure:

- An initial topological graph is built directly as a Markov model, in terms of nodes (states) and edges (state transitions)
- The model is refined through BW algorithm updating the model parameters to better fit the observation sequence
- The discovery of further states is then carried out by a node splitting procedure, observing the evolution in time of the trajectories

**Performance index:** average probability of making the correct prediction for a specific sequence of observations  $O$  of length  $T$  based on  $\lambda$

$$\eta(O, \lambda) = \sqrt[T]{\prod_{t=0}^{T-1} P[\hat{y}_{t+1} = y_{t+1} | y_{0:t}]} = \sqrt[T]{P[O|\lambda]}$$

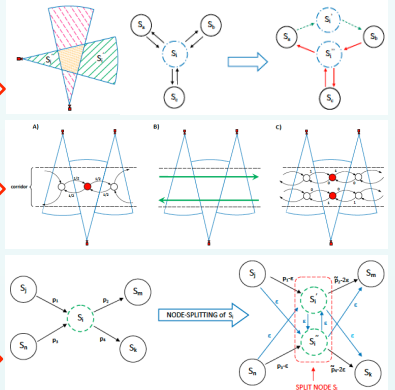
**Topological splitting:** the same observation refers to the target in different positions. Distinct states that relate to the same observation, allows for different future evolution of the trajectory

**Logical Splitting:** past transitions may suggest a preferential direction and allow the estimation of the future state visits (includes topological splitting)

### Orthogonality measure:

	$S_a$	$q_{i+1}$	$S_b$	$S_c$
$q_{i-1}$	$S_a$	0	$P_{a \rightarrow b}$	0
	$S_b$	$P_{b \rightarrow a}$	0	$P_{b \rightarrow c}$
	$S_c$	$P_{c \rightarrow a}$	0	$P_{c \rightarrow c}$

A non-null probability among apparently non related states (i.e. unobserved transitions), is admitted to account for non perfect orthogonality measure and to insert "genetic variability"



## Simulations & Discussion

### Scenario 1:

- Model of corridor where targets move back and forth at constant speed
- Exhibits the need of both topological splitting and logical splitting
- Beginning:  $\eta=0.5$  (same probability of direction change at every step)
- End:  $\eta=1$  (prediction capability is perfect)

### Scenario 2:

- 2D area with random motion (direction and velocity)
- The splitting procedure does not lead to much improvement (from  $\eta=0.25$  to  $\eta=0.3$ ): the random walk of the target does not allow good prediction
- It is not possible to predict the future trajectory based on the observation history: depends only on the current observation

### Scenario 3:

- 2D area with random motion (direction and velocity) with preferential paths
- Situation in between the corridor-like scenario and the 2D random motion
- Partial exploration but same number of distinct observations as Scenario 2
- Prediction capability improves of about 50%: from  $\eta=0.3$  to  $\eta=0.45$

After each splitting step the performance drops considerably due to the incorrect initialization of the matrix  $A$ , however, after a few steps of the Baum-Welch's algorithm, the performance improves becoming higher than before the splitting

