Alternative Frequency-domain Stability Criteria for Networked Systems with Multiple Delays

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1. Motivation

Recent LMI-based stability analysis techniques for NCSs become often untractable for systems of large scale and/or with large delays. Frequency-domain characterizations can be beneficial in this case and can also provide direct ways to solve the corresponding controller design problems.

2. Problem formulation

Discrete-time NCS given by

$$x_{k+1} = Ax_k + Bv_k;\ q_k = Cx_k \qquad (1)$$

interconnected with the time-varying delay block $\mathcal{D}$ given by

$$v_k = q_{k-N_k}. \qquad (2)$$

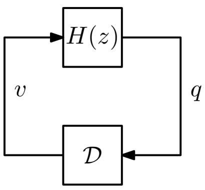

Figure 1: Delay system. $H(z) = C(zI - A)^{-1}B$.

Problem: Determine computationally tractable frequency-domain stability conditions for the delay system (1)-(2) with $N_k \in [m, M] \cap \mathbb{N}$, $k \in \mathbb{N}$.

3. The ℓ_2 gain of a varying delay

Definition: The ℓ_2 gain of the system

$$x_{k+1} = f(x_k, N_k, v_k);\ q_k = g(x_k, N_k, v_k)$$

for uncertainties N_k in Υ is given by

$$\sup \frac{\sqrt{\sum_{k=0}^{\infty} \|q_k\|^2}}{\sqrt{\sum_{k=0}^{\infty} \|v_k\|^2}}$$

with initial condition $x_0 = 0$.

Theorem 1 *The ℓ_2 gain of the delay block (2) with $N_k \in [m, M] \cap \mathbb{N}$, $k \in \mathbb{N}$ is equal to $\sqrt{M - m + 1}$.*

4. Stability of the delay system

Theorem 2 *Consider system (1) with A Schur and ℓ_2 gain strictly smaller than $\frac{1}{\sqrt{M-m+1}}$. Then the NCS (1)-(2) with $N_k \in [m, M] \cap \mathbb{N}$ is GAS.*

The ℓ_2 gain of system (1) is equal to the $\mathcal{H}_\infty$-norm $\|H(z)\|_\infty := \sup_{z \in \mathbb{C}, |z|=1} \bar{\sigma}(H(z))$, which can be computed efficiently.

5. Comparison

Example system : $H(z) = \frac{0.1}{z}$

Using previous results [1]:

- if for all $z \in \mathbb{C}$ with $|z| = 1$ $|\frac{H(z)}{1-H(z)}| < \frac{1}{M|z-1|}$, then the system in Fig. 1 is GAS for $N_k \in [0, M] \cap \mathbb{N}$
- Result: GAS is guaranteed for $N_k \in \{0, 1, 2, 3, 4\}$

New result Theorem 2

- GAS for $N_k \in \{0, 1, \ldots, 98\}$
- Significant improvement!

6. Open-loop unstable systems

In this case a transformation (Fig. 2) is used to rewrite system (1)-(2) as the interconnection of $(I - H(z))^{-1}H(z)$ between w and z, and

$$w_k = z_{k-N_k} - z_k. \qquad (3)$$

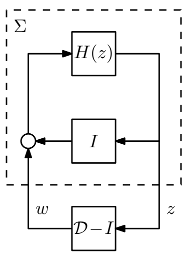

Figure 2: Transformation of delay system.

Theorem 3 *System (1)-(2) with $N_k \in [m, M] \cap \mathbb{N}$, $k \in \mathbb{N}$ is GAS, if $A + BC$ Schur and $\|(I - H(z))^{-1}H(z)\|_\infty$ is strictly smaller than $\frac{1}{1+\sqrt{M-m+1}}$.*

7. NCS with multiple delays

The outputs q are now grouped into L sensor nodes as

$$v_k^i = \mathcal{D}^i q := q^i_{k-N_k^i}, \qquad (4)$$

where $N_k^i \in [m_i, M_i] \cap \mathbb{N}$, $k \in \mathbb{N}$.

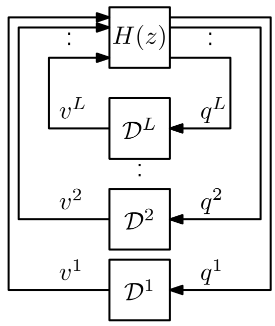

Figure 3: Multiple delay system.

Structured singular value approach:

$$\mathcal{K} := \{\mathrm{diag}(K^1, \ldots, K^L) \mid K^i \text{ invertible},\ i = 1, \ldots, L\}. \qquad (5)$$

Theorem 4 *System (1) interconnected with the delay blocks (4) as in Fig. 3 with time-varying $N_k^i \in [m_i, M_i] \cap \mathbb{N}$, $k \in \mathbb{N}$, is GAS, if A is Schur and*

$$\inf_{K \in \mathcal{K}} \|K \mathrm{diag}(\sqrt{M_1 - m_1 + 1} I_{n_1}, \ldots, \sqrt{M_L - m_L + 1} I_{n_L}) H(z) K^{-1}\|_\infty$$

is strictly smaller than 1.

8. Conclusions

Alternative frequency-domain criteria for stability analysis of delay systems:

- Tractable for large scale systems and large delays
- Easily deal with multiple delays
- Useful for controller synthesis

References

[1] Kao, C. and Lincoln, B. (2004). *Simple stability criteria for systems with time-varying delays.* Automatica, 40(8), 1429- 1434.