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GENERAL FRAMEWORK USING AFFINE TRANSFORMATIONS TO FORMATION CONTROL DESIGN

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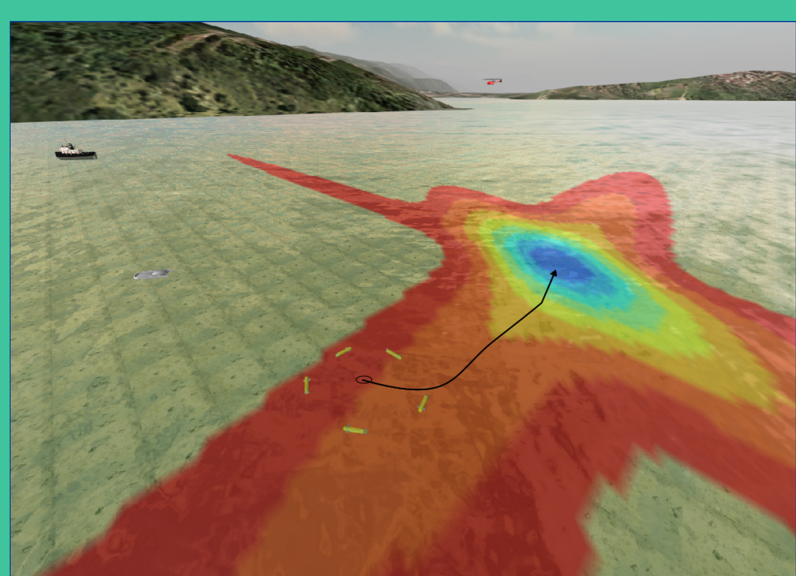
Main Objective

Coordinated Control of a Fleet Formation of AUVs (autonomous underwater vehicles) under communication constraints to achieve a source seeking.



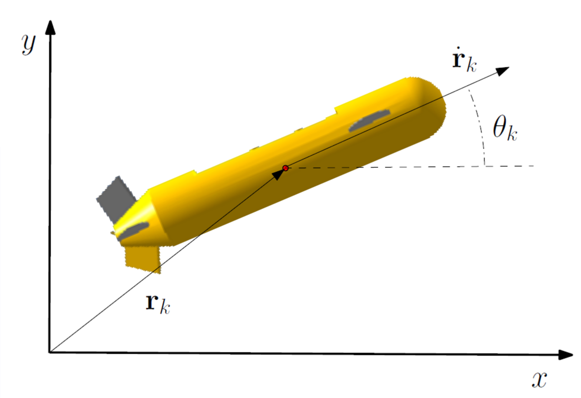
- Non-linear systems representing AUVs
- Novel framework for a larger class of formations
- Cooperative control law based on consensus algorithms
- Uniform distribution
- Communication Constraints

Model of the agents

Consider a set of N agents (vehicles) modeled by a kinematic unicycle fitting with model properties subject to a simple non-holonomic constraint.

$(x_k, y_k)^T$ is the position vector of the agent k
θ_k is the heading angle
u_{1k}, u_{2k} are the control inputs

$$\begin{aligned} \dot{x}_k &= v_k \cos\theta_k \\ \dot{y}_k &= v_k \sin\theta_k \\ \dot{v}_k &= u_{1k} \\ \dot{\theta}_k &= u_{2k} \end{aligned}$$



Problem Formulation

The objective is for the agents to follow different velocities and to stabilize to different formations in a cooperative way.

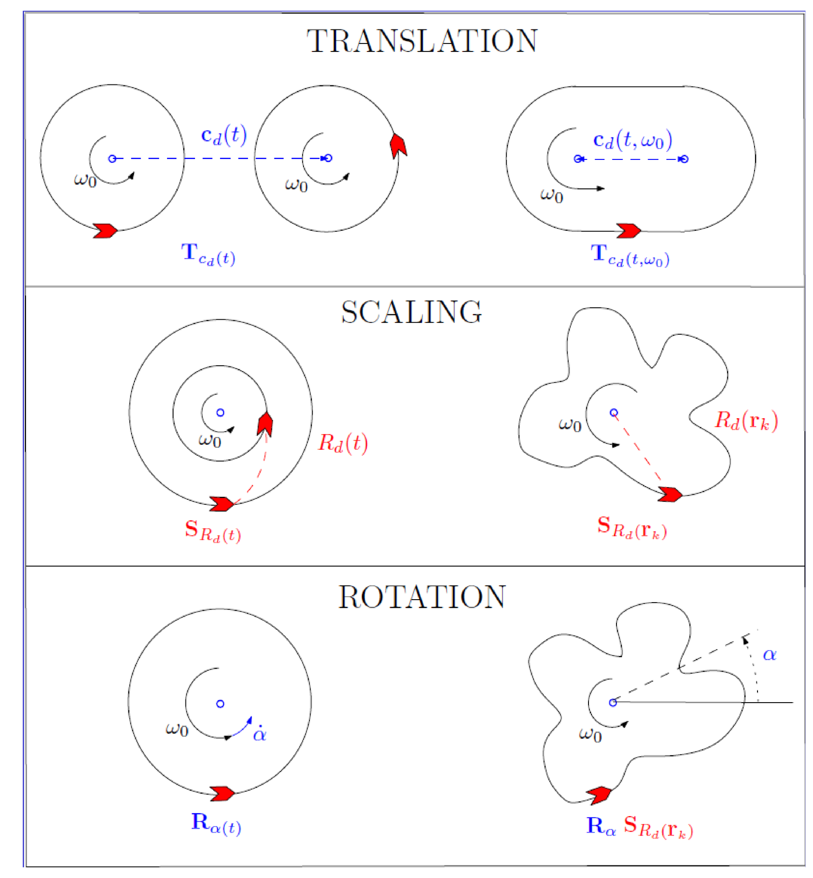
Affine Transformations

Transformation matrices

$$\mathbf{T_C} = \begin{pmatrix} 1 & 0 & c_x \\ 0 & 1 & c_y \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{S} = \begin{pmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{R}_\alpha = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



Sequence of transformations

The main idea is to define any trajectory as a sequence of these transformations applied to a constant vector $\mathbf{r}_0$ expressed in the homogeneous coordinates framework. In the sequel, we consider affine transformations generated by a combination of the basic ones which can be defined as follows:

$$\mathbf{G} = \prod_{i,j,k}^{I,J,K} \mathbf{S}^i \mathbf{R}_\alpha^j \mathbf{T}_c^k$$

Homogeneous coordinates

The homogeneous coordinates of a vector $z \in \mathbb{R}^2$ can simply be defined as the new vector $\mathbf{z}^H = (z_x, z_y, 1)^T$

Position vector in homogeneous coordinates

$$\mathbf{r}_k = \begin{pmatrix} x_k \\ y_k \\ 1 \end{pmatrix}$$

Control Design

The objective can be expressed as $\dot{\mathbf{r}}_k = \dot{\mathbf{G}}\mathbf{r}_0 = \dot{\mathbf{G}}\mathbf{G}^{-1}\mathbf{r}_k$

Theorem

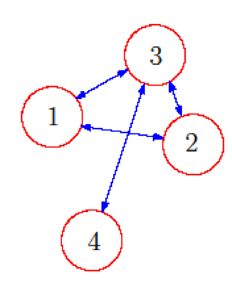
The control law

$$\begin{aligned} u_{1k} &= -\kappa v_k + \frac{1}{v_k}\dot{\mathbf{r}}_k^T \dot{\mathbf{G}}\mathbf{G}^{-1}\dot{\mathbf{r}}_k + \\ &\quad + \frac{1}{v_k}\dot{\mathbf{r}}_k^T \left(\ddot{\mathbf{G}}\mathbf{G}^{-1} + \dot{\mathbf{G}}(\dot{\mathbf{G}^{-1}}) + \kappa\dot{\mathbf{G}}\mathbf{G}^{-1}\right)\mathbf{r}_k \\ u_{2k} &= \frac{1}{v_k^2}\dot{\mathbf{r}}_k^T \mathbf{R}^{*T}\dot{\mathbf{G}}\mathbf{G}^{-1}\dot{\mathbf{r}}_k + \\ &\quad + \frac{1}{v_k^2}\dot{\mathbf{r}}_k^T \mathbf{R}^{*T}\left(\ddot{\mathbf{G}}\mathbf{G}^{-1} + \dot{\mathbf{G}}(\dot{\mathbf{G}^{-1}}) + \kappa\dot{\mathbf{G}}\mathbf{G}^{-1}\right)\mathbf{r}_k \end{aligned}$$

makes all the agents converge to the curve defined by the transformation $\mathbf{G}$ applied to the constant vector $\mathbf{r}_0$.

The condition $v_k \neq 0$ is satisfied to avoid the singular point.

Communication Graph and Laplacian matrix



$$\mathbf{L} = \begin{pmatrix} 2 & -1 & -1 & 0 \\ -1 & 2 & -1 & 0 \\ -1 & -1 & 3 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix}$$

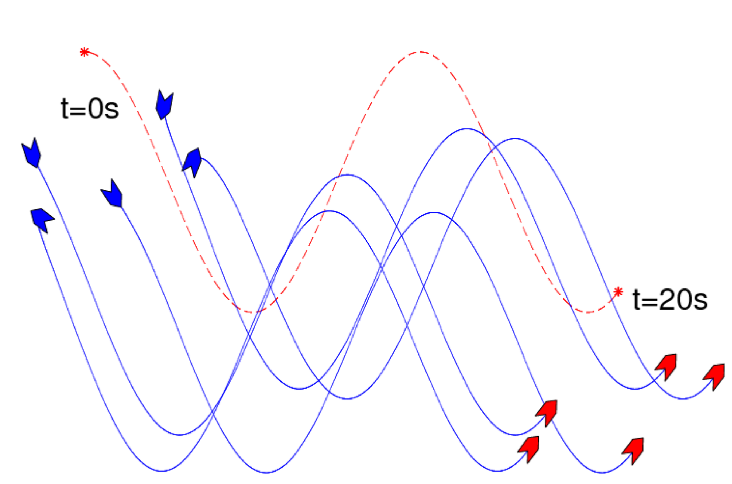
Cooperative Control Design

Theorem

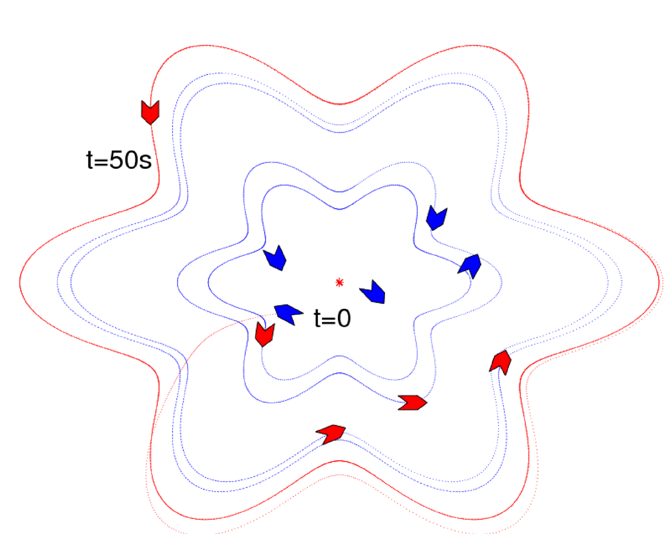
The control law

$$\begin{aligned} u_{1k} &= -\kappa_1 v_k + \kappa_1 \frac{\omega_0}{v_k}\dot{\mathbf{r}}_k^T \mathbf{R}^* \mathbf{r}_k - \kappa_2 \mathbf{L}_k \mathbf{v} \\ u_{2k} &= \omega_0 + \kappa_1 \frac{\omega_0}{v_k^2}\dot{\mathbf{r}}_k^T \mathbf{r}_k \\ U(\theta) &= -\frac{K}{N}\sum_{m=1}^{\lfloor N/2 \rfloor} \frac{1}{2m^2}\mathbf{B}_m^T \overline{\mathbf{L}}\mathbf{B}_m \end{aligned}$$

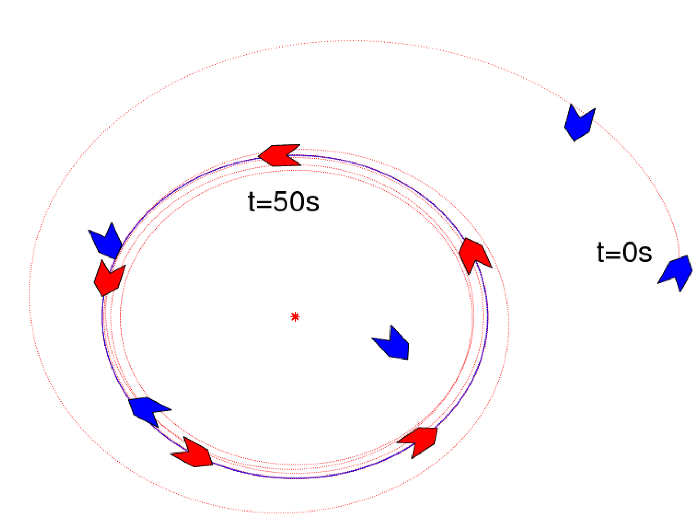
makes all the agents converge to a circular formation centered at the origin and the direction of the rotation defined by the sign of ω_0. Moreover, if the communication graph G is d0-circular, the radius of the circle is obtained through a consensus algorithm applying to the velocities of the agents and the uniform distribution of the agents along the circle is achieved.



Velocity Tracking



Formation Control



Cooperative Circular Formation Control

Conclusions

- General framework to formation control of a fleet of AUVs
- Affine transformations: translation, rotation and scaling
- Time-varying velocity reference
- Cooperative control law to achieve the circular formation
- Uniform distribution of the agents along the circle
- Consensus algorithm and the potential function designed taking into account the communication constraints

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