Distributed Estimation over Unknown Fading Channels

Alain Y. Kibangou

GIPSA-LAB/University Joseph Fourier/ CNRS, Grenoble, France

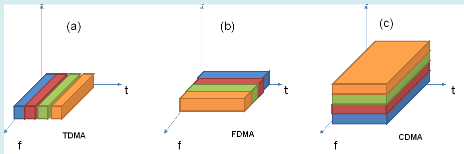
Problem setting

- Sensing model: $\mathbf{x}^{(k)} = \boldsymbol{\theta} + \mathbf{n}^{(k)}, \quad k = 1, \cdots, K$
  - $\mathbf{x}^{(k)}$: measurement at the kth node
  - $\boldsymbol{\theta} \in \Re^N$: real-valued unknown vector of parameters
  - $\mathbf{n}^{(k)}$: observation noise vector at the kth node
- LS estimator: $\hat{\boldsymbol{\theta}} = (1/K)\sum_{k=1}^{K} c,\ \hat{\boldsymbol{\theta}}_k(0) = \mathbf{x}^{(k)}$
- Distributed estimation: $\hat{\boldsymbol{\theta}}_k(t) \to \hat{\boldsymbol{\theta}}$.

$$\hat{\boldsymbol{\theta}}_k(t+1) = f\left(\hat{\boldsymbol{\theta}}_t, \varphi(\hat{\boldsymbol{\theta}}_j(t), \beta_{k,j}, \tau_{k,j})\right),\ j \in \mathcal{K}_k$$

- $\varphi(.)$: Data detector function.
- $\mathcal{K}_k = \{j | \{k,j\} \in \mathcal{E}\}$
- $\mathcal{G} = \{\mathcal{K}, \mathcal{E}\}$: Undirected connected graph representing the communication graph.
- $\mathcal{K} = \{1, \cdots, K\}$ and $\mathcal{E}$ denote respectively the node set and the edge set, where each edge $\{i,j\} \in \mathcal{E}$ is an unordered pair of distinct nodes.

- Additive noise, Fading: $\beta_{l,k}$
- Delay and asynchronism: $\tau_{l,k}$



- TDMA: can induce a latency.
- FDMA: crucial scalability issue due to limited bandwidth.
- CDMA: requires orthogonal waveforms and perfect synchronization.

Data detection

- 1-D model:

$$y_j^{(l)} = y^{(l)}(t)|_{t=jT_s} = \sum_{k \in \mathcal{K}_l} s_{j,k} h_k^{(l)}$$

where $h_k^{(l)} = \beta_{l,k} g_k(t - jT_s - \tau k, l)|_{t=jT_s}$.

$$\Updownarrow$$

$$\mathbf{y}^{(l)} = \mathbf{S}^{(l)} \mathbf{h}^{(l)}$$

  - Source separation problem with several sources and a single receiver.
  - Can be solved by using linear Space-Time Block Codes requiring a perfect channel state information.

- 2-D model: Use of a single spreading sequence

$$y_{j,i}^{(l)} = y^{(l)}(t)|_{t=jT_s+iT_c} = \sum_{k \in \mathcal{K}_l} s_{j,k} h_{i,k}^{(l)}$$

where $h_{i,k}^{(l)} = \beta_{l,k} c_{i,k} g_k(t - jT_s - iT_c \tau_{k,l})|_{t=jT_s+iT_c}$.

$$\Updownarrow$$

$$\mathbf{Y}^{(l)} = \mathbf{S}^{(l)} \mathbf{H}^{(l)T} = \sum_{k \in \mathcal{K}_l} \mathbf{S}_{.k}^{(l)} \circ \mathbf{H}_{.k}^{(l)}$$

  - Non unique bilinear decomposition $\Rightarrow$ blind data detection not possible.
  - Can be solved by using a learning sequence $\Rightarrow$ additional communication overload.

- 3-D model: Use of two spreading sequences

$$y_{j,q,i}^{(l)} = y^{(l)}(t)|_{t=jT_s+iT_c+qT_f} = \sum_{k \in \mathcal{K}_l} s_{j,k} b_{q,k} h_{i,k}^{(l)}$$

where $h_{i,k}^{(l)} = \beta_{l,k} c_{i,k} g_k(t - jT_s - iT_c - qT_f \tau_{k,l})|_{t=jT_s+iT_c+qT_f}$

$$\Updownarrow$$

$$\mathbb{Y}^{(l)} = \sum_{k \in \mathcal{K}_l} \mathbf{S}_{.k}^{(l)} \circ \mathbf{B}_{.k}^{(l)} \circ \mathbf{H}^{(l)}$$

  - Essentially unique trilinear decomposition $\Rightarrow$ blind data detection is possible.
  - Powerful scheme for data detection over unknown fading channels.

Uniqueness of the 3-D model

- Essential uniqueness: each factor matrix can be determined up to column scaling and permutation.
- Observations:
  - Node-wise independent fading and independent design of the pulse-shape filters $\Rightarrow \mathbf{H}^{(l)}$ is full rank with high probability.
  - Independence of the spreading sequence $\{b_{q,k}\}$, not restricted to belong to a finite alphabet, $\Rightarrow \mathbf{B}^{(l)}$ is full rank with high probability.
  - Columns of $\mathbf{S}^{(l)}$ must be pairwise independent.

$$\mathbf{S}^{(l)} = \begin{pmatrix} \mathbf{z}_1 & \cdots & \mathbf{z}_{K_l} \\ \hat{\boldsymbol{\theta}}_1(0) & \cdots & \hat{\boldsymbol{\theta}}_{K_l}(0) \end{pmatrix}$$

with linearly independent vectors $\mathbf{z}_q$.

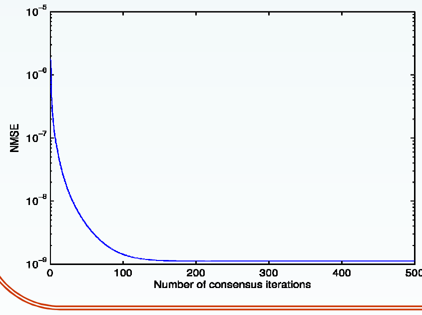
- Condition:

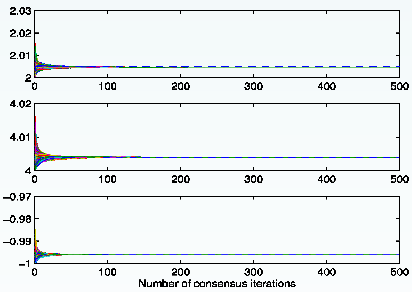
$$Q \geq \max(K_l), \quad I \geq \max(K_l).$$

Q and I being the length of the spreading sequences.

Simulation results

- Network: Grid of 10×10 nodes.
- Performance evaluation by means of NMSE.





Conclusion

- New scheme for distributed estimation over Non orthogonal channels with unknown fading.
- Local data modulated by doubly spread waveforms $\Rightarrow$:
  - Nodes can communicate simultaneously.
  - The received data exhibit a trilinear structure that can be used for separating the data.
- Sufficient conditions ensuring identifiability or uniqueness of the detected data.
- The detection error induced by the channel noise can degrade the overall estimation process.
- A further analysis of the performance of the proposed data detection is still under investigation.

Contact:

alain.kibangou@ujf-grenoble.fr

NeCS Networked Controlled System Team · gipsa-lab · cnrs · Université Joseph Fourier Sciences. Technologie. Santé · FeedNetback co-design for networked control systems