Wireless Routing on a Poisson Point Process

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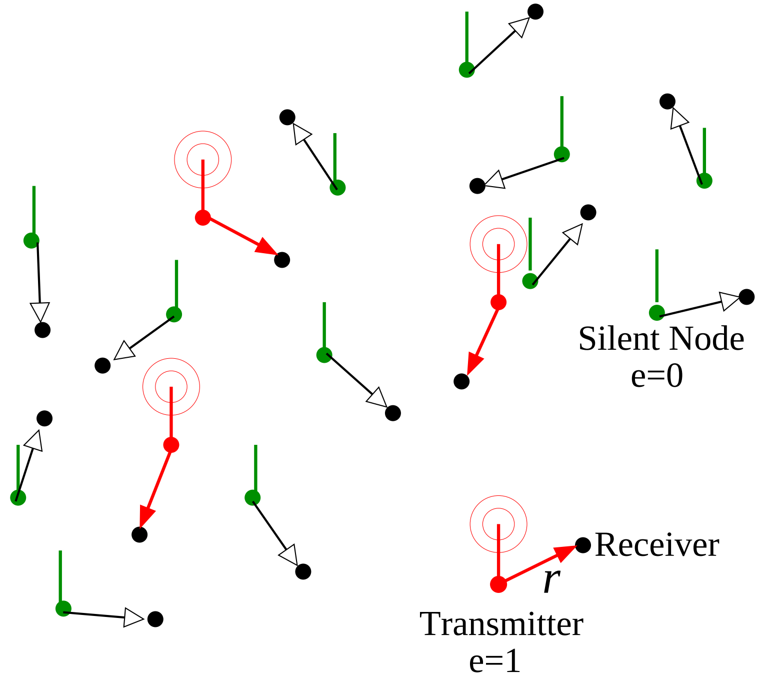
SUMMARY OF THE LECTURE

- **Stochastic Geometry and Wireless Networks**
  - **MAC**
  - **Routing**
  - **Nature of results**
- **Wireless Routing**
  - **MAC**
    - **Aloha in MANETs**
  - **Routing**
    - **Delays on Long Routes**

REFERENCES

- **General Framework: NOW Monograph, 2009 Stochastic Geometry and Wireless Networks F.B. and B. Blaszczyszyn**
  - **Volume I (Theory), file FnT1.pdf: http://hal.inria.fr/inria-00403039**
  - **Volume II (Applications), file FnT2.pdf: http://hal.inria.fr/inria-00403040**
- **This lecture**
  - **Local Delays**
    (joint work with B. Blaszczyszyn, Infocom'10)
  - **Opportunistic Routing**
    (joint w. with B. Blaszczyszyn and O. Mirsadeghi, AAP to appear)

ALOHA ON A POISSON BIPOLAR MANET



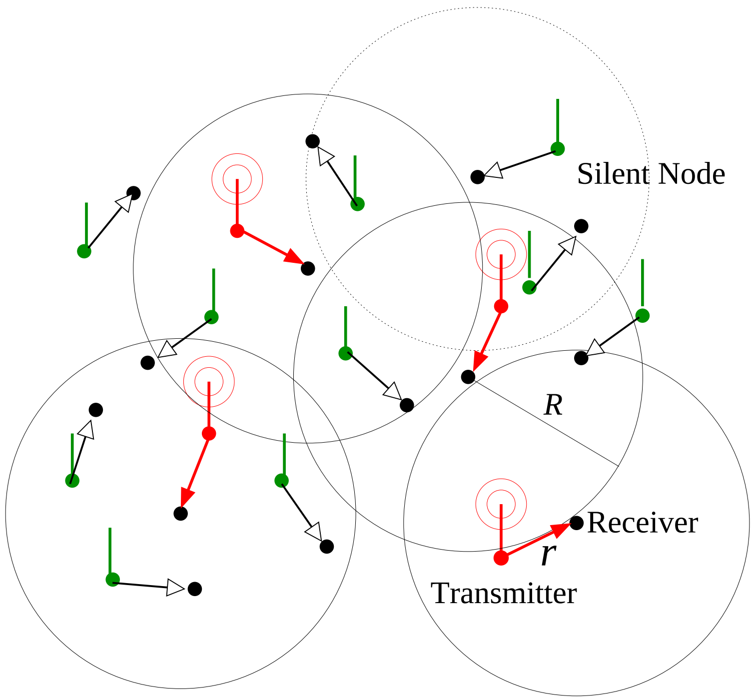
Each node has its receiver at a fixed distance r in some uniform direction.

- **Nodes form a Poisson p.p. in the Euclidean plane**
- **Each node uses Aloha**
- **Transmission depends on SINR at the receiver e.g.**
  - **CBR (outage) model: success if SINR $>$ T**
  - **VBR (adaptive coding) model: rate = C ln(1+SINR)**

QUESTIONS ON THE BIPOLAR MODEL

- **Spatial Averages for**
  - **Success of transmission in the outage case**
  - **Time to transmit a packet in the outage case**
  - **Rate of transmission in the VBR case**
  - **Number of packets $\times$ meters transmitted per second**
- **Optimization of the latter w.r.t. the network/protocol parameters**
- **Key stochastic geometry object: the interference field seen as a Poisson Shot Noise**

CSMA IN A POISSON BIPOLAR MANET



Hard core exclusion p.p. built on a Poisson p.p.
Matérn p.p., Gibbs p.p., Determinental p.p.

– **Carrier Sensing makes sure that no active receiver has an active transmitter withing CS range R**

– **Transmission depends on SINR at the receiver**

- **Probability for a node to access the shared medium**
- **Same questions as above, but on hard core p.p.**

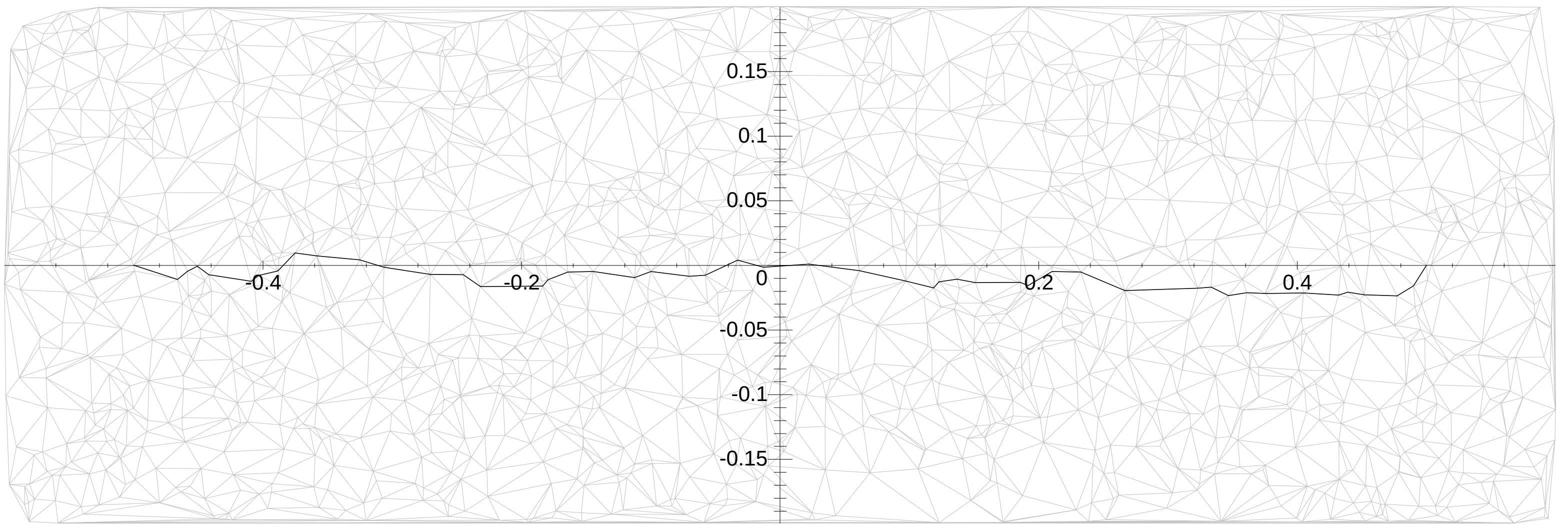
ROUTING MOTIVATIONS

On a MANET with randomly placed nodes,

- **Optimal routes**
- **Greedy routes**
- **Wireless routes**

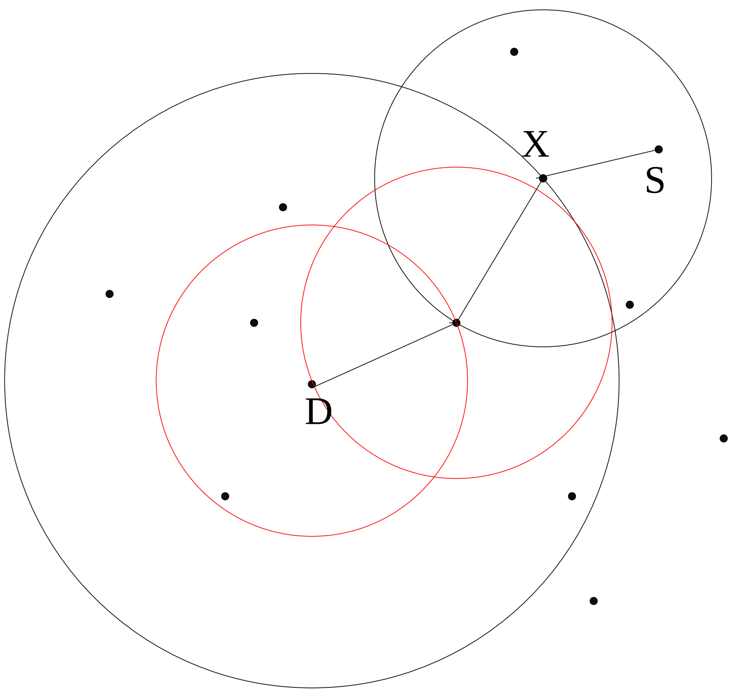
can be seen as random geometric objects (closed sets)

OPTIMAL ROUTES ON A POISSON POINT PROCESS



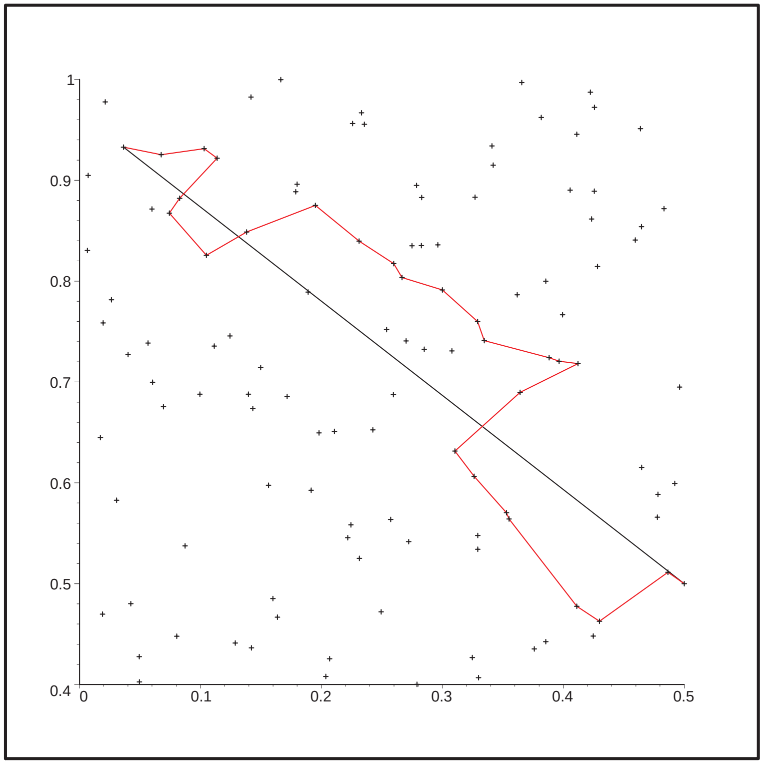
Euclidean shortest path on the Poisson–Delaunay graph.

GREEDY ROUTES ON A POISSON POINT PROCESS



- **Nearest neighbor geographic routing on the Poisson p.p.**
- **Algorithm**:
  The next hop on the route from any node X of the route from S to D is the nearest among the nodes which are closer from D than X.

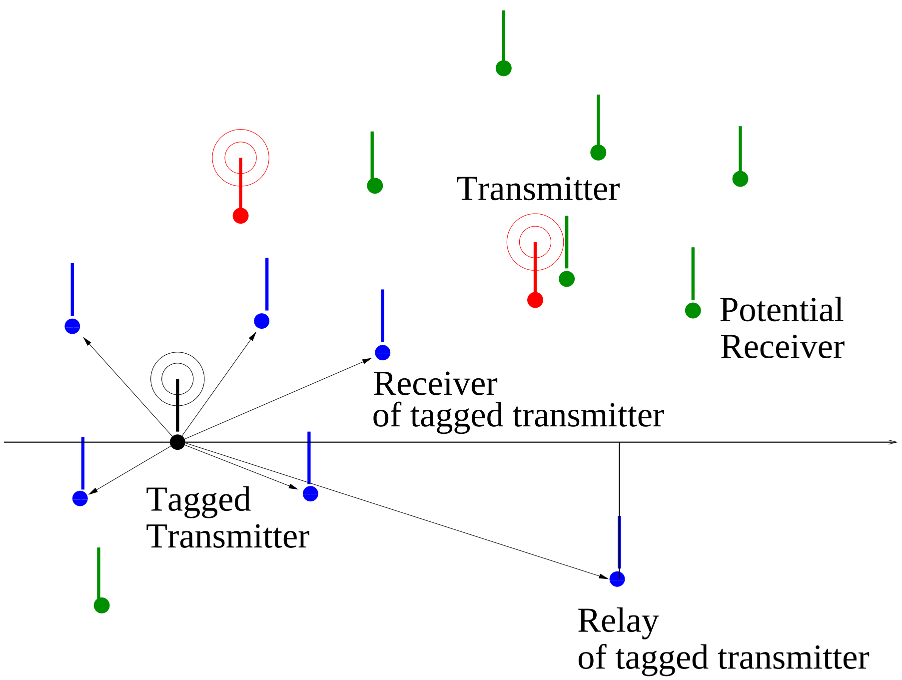
GREEDY ROUTES ON A POISSON POINT PROCESS (*continued*)



Nearest neighbor geographic route on the Poisson p.p.

- **The path can be built based on local information**
- **Edges are between neighbors, which is optimal in most wireless networks**

EXAMPLE OF WIRELESS ROUTES ON A POISSON POINT PROCESS



- **Each node uses Aloha to split the Poisson MANET into transmitters and potential receivers**
- **CBR model: Relay = potential receiver with maximal progress towards destination**
- **Slotted time: Routes are Space–time random structures**

MAIN QUESTIONS ON ROUTES ON A POINT PROCESS

- **Convergence of routing algorithms**
- **Scaling laws of routes (length, overhead)**
- **Space averages: Local geometry of the route: (length of edge/hop, local delay, collisions)**
- **Existence of route averages**
- **Comparison of route averages and space averages**

NATURE OF RESULTS

– **Quantitative**: full analogy with queueing theory

– **Qualitative**: phase transitions

  * **Connectivity–Coverage**
  * **Packet latency on long routes**
  * **Power control feasibility**

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SPATIAL ALOHA PRINCIPLES

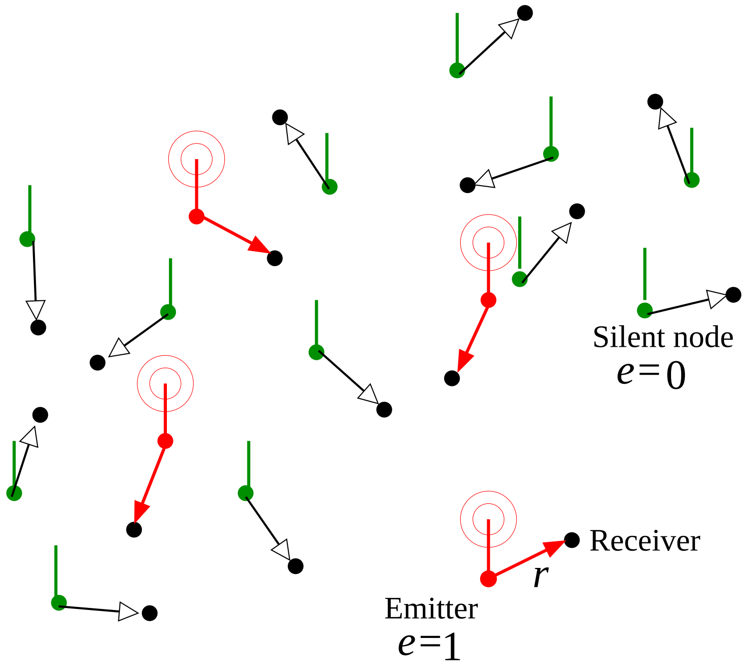
- **Data: wireless network made of nodes (potential transmitters) which form some realization of a Poisson point process in the Euclidean plane.**
- **Slotted version of Spatial Aloha:**
  - **Each node tosses a coin with bias p (MAP) to be allowed to access the medium**
  - **This creates a random exclusion area around each location**
  - **Success of transmission is decided in function of the SINR at the receiver of each effective transmitter.**

SINR MODEL FOR SPATIAL ALOHA

- $\Phi = \{X_i, (F_{i,.})\}$ **independently marked PPP**
- $e_i \in \{0, 1\}$**: right for** i **to access medium in current slot**
- $I_{\Phi_1}(y) = \sum_{i \neq 0} F_{i,y} e_i / l(|y - X_i|)$**: 'filtered' interference at** y**,**
- **Node at** 0 **active in slot can be received by that located at** y **iff**

$$\frac{F_{0,y}/l(|y|)}{W + I_{\phi_1}(y)} \geq T.$$

BIPOLAR ALOHA NETWORK



Each node has its receiver at some fixed distance r in some uniformly and independently sampled direction.

PROBABILITY OF TRANSMISSION SUCCESS FOR GIVEN RANGE: M/M CASE

- **Notation**
  - Φ **Poisson with intensity** λ
  - e_i **i.i.d. with** $I\!\!P(e_i = 1) = p \Rightarrow \Phi_1$ **Poisson with intensity** λp**.**
  - **Rayleigh fading:** $F_{i,j}$ **i.i.d. exponential with mean** μ^{-1}
  - $\mathcal{L}_A$**: Laplace transform of the random variable** $A \geq 0$**.**

PROBABILITY OF TRANSMISSION SUCCESS FOR GIVEN RANGE: M/M CASE (*continued*)

- **The Laplace functional of a p.p. Φ is**

$$\mathcal{LF}_\Phi(f) = \mathbf{E}\left[e^{-\int_{\mathbb{R}^d} f(x)\,\Phi(dx)}\right],$$

where f runs over the set of all non-negative functions on $\mathbb{R}^d$.

- **PROPOSITION The Laplace functional of the Poisson p.p. of intensity λ is**

$$\mathcal{LF}_\Phi(f) = e^{-\int_{\mathbb{R}^d}(1-e^{-f(x)})\lambda dx}.$$

The Laplace transform of the interference I_{Φ_1} is

$$\mathcal{L}_{I(y)}(t) = \exp\left\{-2\pi\lambda\int_0^\infty r\Big(1-\mathcal{L}_F\big(t/l(r)\big)\Big)\,dr\right\},$$

where $\mathcal{L}_F(t)$ is the Laplace transform of the fading.

PROBABILITY OF TRANSMISSION SUCCESS FOR GIVEN RANGE: M/M CASE (*continued*)

- **LEMMA [IEEE Tr. IT 06] For Rayleigh fading, the probability of coverage of an extra receiver at distance r from some transmitter is**

$$\begin{aligned} p_r &= \mathbb{P}(F \geq Tl(r)(W + I_{\Phi_1})) = \mathcal{L}_W\left(\mu T l(r)\right) \mathcal{L}_{I_{\Phi_1}}\left(\mu T l(r)\right) \\ &= \mathcal{L}_W\left(\mu T l(r)\right) \exp\left\{ -2\pi\lambda p \int_0^\infty \frac{u}{1 + l(u)/(Tl(r))} du \right\}. \end{aligned}$$

- **In the power law attenuation case with $W \equiv 0$,**

$$p_r = \exp(-\lambda p r^2 T^{2/\beta} K(\beta)),$$

where

$$K(\beta) = \frac{2\pi\Gamma(2/\beta)\Gamma(1 - 2/\beta)}{\beta} = \frac{2\pi^2}{\beta \sin(2\pi/\beta)}.$$

BEST MAP GIVEN RANGE

- **Question: what value of p maximizes the spatial density of successful transmissions in the bipolar model with dist. r.**
- **LEMMA [IEEE IT 06] maximization of $\lambda p_r(\lambda)$ spatial density of successful access to channel w.r.t. λ.**

$$\lambda_{\max} = \frac{1}{2\pi \int_0^\infty \frac{u}{1+l(r)/(Tl(u))} du}$$

and

$$\lambda_{\max} p_r(\lambda_{\max}) = e^{-1} \lambda_{\max} \mathcal{L}_W \left(\frac{\mu T}{l(r)} \right)$$

BEST MAP GIVEN RANGE (*continued*)

- **In the power law attenuation case,**
  - **Optimal spatial intensity of channel access**

$$\lambda_{\max} = \frac{1}{r^2 T^{2/\beta} K(\beta)}$$

  - **Optimal spatial intensity of successful transmissions**

$$\lambda_{\max} p_r(\lambda_{\max}) = \frac{1}{r^2 T^{2/\beta} e K(\beta)}$$

PROBABILITY OF TRANSMISSION SUCCESS FOR GIVEN RANGE: M/G CASE

- **Assumptions**
  - Φ **Poisson with intensity** λ**;**
  - $F_{i,j}$ **i.i.d. with general distribution with mean** μ^{-1}**;**
  - **Power law attenuation model;** $W \equiv 0$**.**

PROBABILITY OF TRANSMISSION SUCCESS FOR GIVEN RANGE: M/G CASE (*continued*)

- **Fourier representation of the probability of coverage**

LEMMA [JSAC, 2009] If

- **–** F **has a finite first moment and a square integrable density;**
- **– Either** I_1 **or** W **admit a density which is square integrable, then**

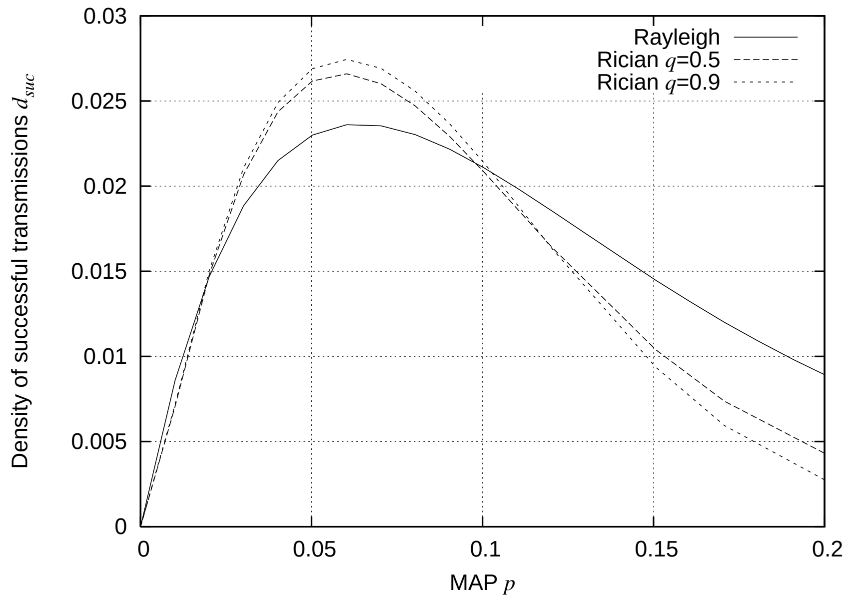
the probability of a successful transmission at distance r **is**

$$p_r(\lambda p) \;=\; \int_{s=-\infty}^{\infty} \mathcal{L}_{I_1}\left(2i\pi l(r)Ts\right) \mathcal{L}_W\left(2i\pi l(r)Ts\right) \frac{\mathcal{L}_F(-2i\pi s)-1}{2i\pi s} ds\,.$$

with

$$\mathcal{L}_{I_1}(s) = \exp\left\{-2\pi\lambda p \int_0^{\infty} r\left(1-\mathcal{L}_F\left(s/l(r)\right)\right) dr\right\},$$

IMPACT OF FADING



Density of successful transmissions in Aloha in function of p; in the Rayleigh and the Rician (with $q = 1/2$ and $q = .9$) fading cases; $\lambda = r = 1$, $W = 0$, and $T = 10$dB, $\beta = 4$.

THROUGHPUT M/M Case

- **Shannon throughput (bit-rate) of the channel from transmitter x_i to its receiver y_i by**

$$\mathcal{T}_i = \log(1 + \mathbf{SINR}_i)\,.$$

- **Theorem [JSAC, 2009] For Rayleigh fading and for $l(r) = (Ar)^\beta$,**

$$\tau = \frac{\beta}{2}\int_0^\infty e^{-\lambda_1 K(\beta) r^2 v} \frac{v^{\frac{\beta}{2}-1}}{1+v^{\frac{\beta}{2}}} \mathcal{L}_W\left(\mu (Ar)^\beta v^{\beta/2}\right) dv$$

and

$$\mathcal{L}_{\mathcal{T}}(s) = \frac{\beta s}{2}\int_0^\infty \left(1 - e^{-\lambda_1 K(\beta) r^2 v} \mathcal{L}_W\left(\mu (Ar)^\beta v^{\beta/2}\right)\right) \frac{v^{\frac{\beta}{2}-1}}{\left(1+v^{\frac{\beta}{2}}\right)^{1+s}} dv\,,$$

THROUGHPUT M/G CASE

- **For the model with general fading F**

$$\tau = \int_0^\infty \int_{-\infty}^\infty \mathcal{L}_{I^1}\left(2i\pi vsl(r)\right) \mathcal{L}_W\left(2i\pi vsl(r)\right) \frac{\mathcal{L}_F(-2i\pi s) - 1}{2i\pi s(1+v)} ds dv$$

and

$$\mathcal{L}_{\mathcal{T}}(s) = 1 - s \int_0^\infty \int_{-\infty}^\infty \mathcal{L}_{I^1}\left(2i\pi vsl(r)\right) \mathcal{L}_W\left(2i\pi vsl(r)\right) \frac{\mathcal{L}_F(-2i\pi s) - 1}{2i\pi s(1+v)^{1+s}} ds dv \, .$$

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TIME-SPACE SINR GRAPH ON A POISSON P.P.

- **Static Poisson point process $\widetilde{\Phi}$ of node locations with varying marks:**
  - $\{e_i^n\}$**: i.i.d. (time dependent Aloha). Split: Φ_1^n: transmit, Φ_0^n: silence.**
  - $\{F_{i,j}^n\}$ **(time dependent fading)**
  - $\{W_j^n\}$ **(time dependent thermal noise)**
- **Interference at x at time n for transmitter i:**
  $I^i_{\Phi_1^n}(x) = \sum_{l \neq i} F^n_{l,x} e^n_l / l(|x - X_l|)$
- **Receiver X_j captures packet from transm. X_i at time n iff**

$$\frac{F^n_{i,j}/l(||X_i - X_j||)}{W^n_j + I^i_{\phi_1^n}(X_j)} \geq T.$$

TIME-SPACE SINR GRAPH ON A POISSON P.P. (*continued*)

- **TIME-SPACE SINR Graph:** $\mathbb{G}_{\text{SINR}}$
  - **Nodes:** (X_i, n), $X_i \in \Phi$, $n \in \mathbb{Z}$;
  - **Directed edges: out-neighbors of** X_i **at time** n:

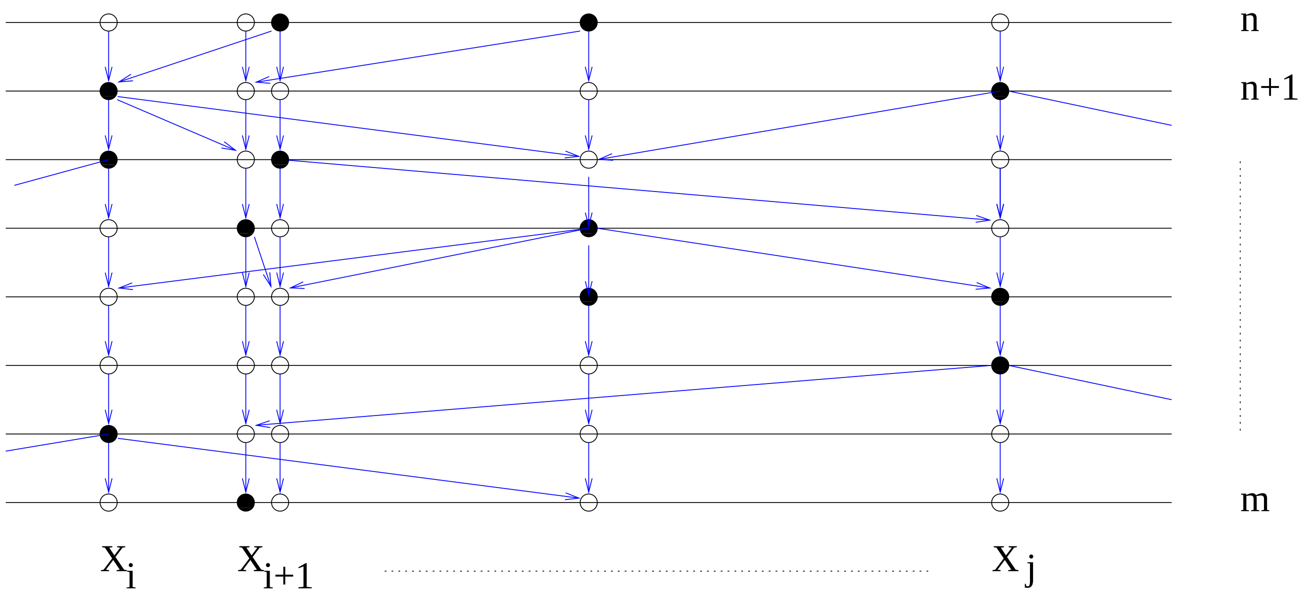
$$V^+(X_i, n) = \begin{cases} \{X_j : X_j \in \Phi_0^n \textbf{ s.t. } X_j \textbf{ captures } X_i \textbf{ at } n\} & \textbf{if } X_i \in \Phi_1^n \\ \{X_i\} & \textbf{otherw.} \end{cases}$$

VARIANTS: FADING SCENARIOS

Slow fading: all $F_{i,j}^n$ are sampled independently for each transmitter-receiver pair and stay constant over time.

Fast fading: all $F_{i,j}^n$ are sampled independently for each time and each transmitter-receiver pair.

THE TIME-SPACE SINR GRAPH



A sample of the $\mathbb{G}_{\mathrm{SINR}}$ graph for a Poisson p.p. on the line, for the fast fading case.

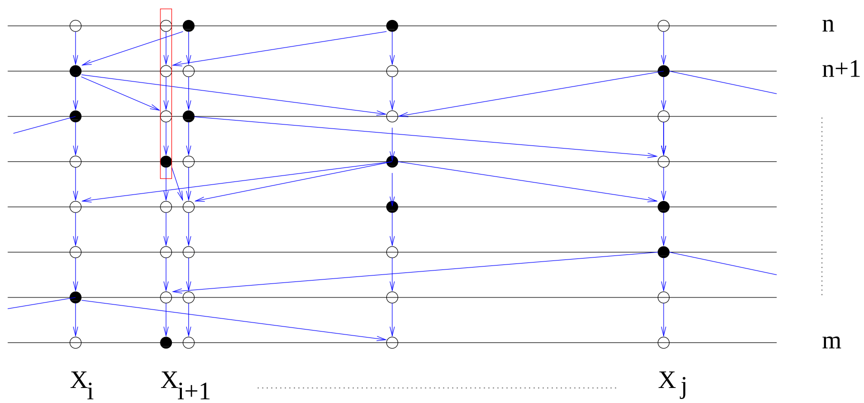
RELAY SELECTION VARIANTS

Nearest receiver relay (NRR): each transmitter X selects the nearest point of Φ as its relay and tries until success.

Any receiver relay (ARR): each transmitter X broadcasts and any point of Φ which is a neighbor of X can be selected as relay (one may take the one maximizing some utility).

LOCAL DELAY

- **Local delay at $X \in \Phi$, $\mathbf{L}(X)$: time to transmit a packet to the next hop under Aloha**
- **Two incarnations:**
  - **Time to transmit to the selected receiver in NRR**
  - **Time to be received by some receiver in ARR**



Local delay for NRR.

LOCAL DELAY (*continued*)

- **Lemma** Under either
  - (a) fast fading and F with unbounded support or
  - (b) W=0

local delays are P_0^{Φ} a.s. finite provided $0 < p < 1$.

- **Proof**:
  Conditional on the static elements of the network (Φ),
  L is a geometric random variable.

MEAN LOCAL DELAYS UNDER NRR

- **Lemma** **[Infocom'10]**
  Assume fast Rayleigh fading and $\{W(n)\}$ i.i.d. Under NRR,

$$\mathbf{E}^0[\mathbf{L}] = 2\pi \int_{r>0} \frac{r\exp(-\pi r^2)}{p(1-p)\mathcal{L}_W(\mu l(r)T)} e^{\pi \int_{v>0} \frac{\lambda p T l(r)}{l(v)+(1-p)Tl(r)} v dv + \int_{\theta=-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{v>2r\cos\theta} \frac{\lambda p T l(r)}{l(v)+(1-p)Tl(r)} v dv d\theta} dr$$

NRR EXAMPLES: Noise Limited Networks

- **In Noise Limited Networks (interference is perfectly cancelled**
  A thermal noise bounded from below by a positive constant leads to infinite mean local delays!

NRR EXAMPLES: Interference Limited Networks

- **Here, no thermal noise, Rayleigh fading**

$$J(w,b) = \int_{\theta=-\pi/2}^{\pi/2} \int_{u>(2\cos\theta)^2 w^{-1/b}} \frac{1}{1+u^b} du d\theta \, .$$

  - **If** $p \neq 0$ **and**

$$\frac{p}{(1-p)^{1-\frac{2}{\beta}}} T^{\frac{2}{\beta}} \left(\frac{K(\beta)}{2} + J(T(1-p), \frac{\beta}{2}) \right) < \pi,$$

    then $\mathbf{E}^0[\mathbf{L}] < \infty$**;**

  - **If**

$$\frac{p}{(1-p)^{1-\frac{2}{\beta}}} T^{\frac{2}{\beta}} \left(\frac{K(\beta)}{2} + J(T(1-p), \frac{\beta}{2}) \right) > \pi,$$

    then $\mathbf{E}^0[\mathbf{L}] = \infty$**.**

NRR EXAMPLES: Interference Limited Networks (*continued*)

- **Bounds: let**

$$\Theta(p,T,\beta) = \frac{p}{(1-p)^{1-\frac{2}{\beta}}} T^{\frac{2}{\beta}} \frac{K(\beta)}{\pi}.$$

  – **If** $p \neq 0$ **and**

$$\Theta(p,T,\beta) < 1,$$

  then $\mathbf{E}^0[\mathbf{L}] < \infty$**.**

  – **If**

$$\Theta(p,T,\beta) > 2,$$

  then $\mathbf{E}^0[\mathbf{L}] = \infty$**.**

PHASE TRANSITION

- **Not an artifact of the assumptions:**
  - **The same phenomenon holds true for the case of bounded attenuation**
  - **The phenomenon has an incarnation in several other scenarios like the bipole model or ARR**
- **It is linked to the RESTART phenomenon**

RESTART

- **A file of random size B is to be transmitted over an error prone channel. B has infinite support.**
- $\{A_n\}_{n \geq 1}$**: i.i.d. sequence of channel inter-failure times, independent of B. A_n is light tailed**
- **If $A_1 > B$ (resp. $A_1 \leq B$), the transmission succeeds (resp. fails) at the first attempt.**
- **If the transmission fails at the first attempt, one has to restart the whole file transmission in the second attempt and so on.**

$$N = \inf\{n \geq 1 \; s.t. \; A_n > B\} \quad \textbf{is heavy tailed}.$$

INTERPRETATION OF PHASE TRANSITION

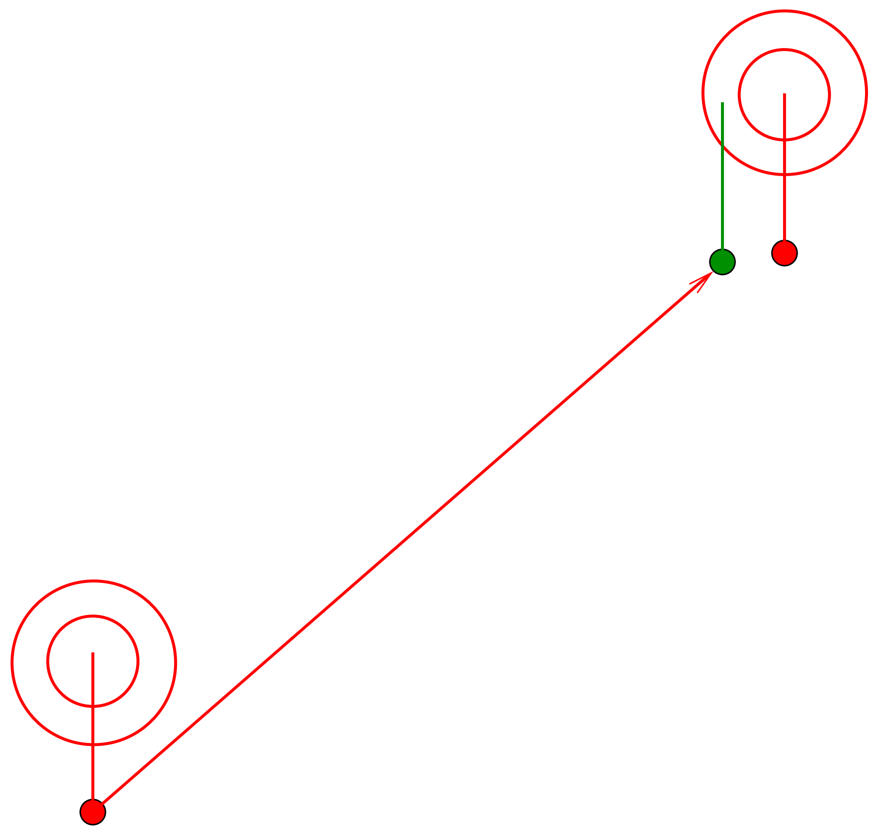
- **Due to contention for wireless medium:**
  - **If the density of receivers is too small;**
    1. **at any given time slot, transmitters compete for too small a set of receivers;**
    2. **each targeted receiver is too far away from its transmitter**
  - **For T and β fixed, stability requires that receivers outnumber potential transmitters by a factor which grows like**

$$\frac{p}{(1-p)^{1-2/\beta}}$$

if this condition is not satisfied, some receivers have too persistent interferers nearby.

INTERPRETATION OF PHASE TRANSITION (*continued*)

■ **A random trap**



IDEA OF PROOF (fixed Poisson receiver model)

$$\mathbf{P}^0[\mathbf{L} > m \mid D = r] = \mathbf{P}^0\left[\cap_{n=1}^{m} \{e_0^n F_0^n < l(r) T(I_1^n + W^n)\}\right],$$

where

- $I_1^n = \sum_{i \neq 0} \frac{e_i^n F_i^n}{l(|X_i - Y|)}$ **is the SN seen by receiver** Y **at time** n**.**
- F_i^n **is the fading from transmitter** X_i **of** Φ **to** Y **at time** n**.**
- W^n **is the thermal noise at** Y **at time** n**.**

Here, Y **is the location of the receiver of node 0.**

IDEA OF PROOF (fixed Poisson receiver model) (*continued*)

- **$\mathcal{G}$: the σ-algebra generated by Φ (without its marks).**
- **Since $\{I^1(n) + W(n)\}$ is i.i.d. conditionally on $\mathcal{G}$:**

$$\begin{aligned}
&\mathbf{P}^0[\mathbf{L} > m \mid D = r] \\
&\quad = \mathbf{P}^0\left[\cap_{n=1}^m \{e_0^n F_0^n < l(r) T (I_1^n + W^n)\} \mid \mathcal{G}\right] \\
&\quad = \mathbf{E}^0\left[(1-p) + p\left(1 - \exp\left(-\mu l(r) T (I_1 + W)\right)\right) \mid \mathcal{G}\right]^m \\
&\quad = \mathbf{E}^0\left[1 - p \exp\left(-\mu l(r) T (I_1 + W)\right) \mid \mathcal{G}\right]^m \\
&\quad = \left[1 - p \mathcal{L}_W(\mu l(r) T) \prod_{i \in \Phi, i \neq 0} \left(1 - p + p \frac{1}{1 + T l(r)/l(|X_i|)}\right)\right]^m
\end{aligned}$$

- **Then sum up w.r.t. m and use Laplace functional for Poisson p.p.**

SIMILAR RESULTS FOR THE ARR CASE

- **Example: noise limited network**

$$E^0[\mathbf{L}] = \frac{1}{p}\sum_m \exp\left(-2\pi\lambda \int_{r>0} (1-(1-(1-p)\mathcal{L}_W(\mu l(r)T))^m)\, r dr\right) .$$

- **A sufficient condition for convergence is that**

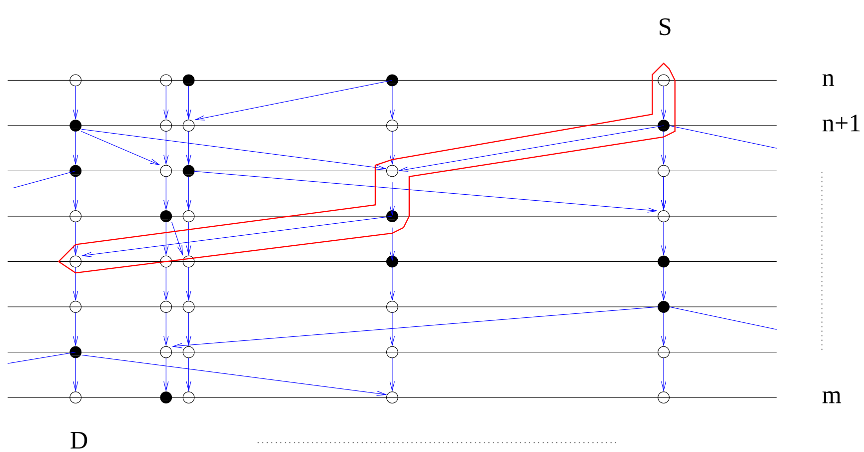
$$\mathcal{L}_W(x) \geq \frac{\kappa}{1-p}\exp\left(-\frac{\pi\lambda}{1+\epsilon}\left(\frac{x}{\mu T A^\beta}\right)^{2/\beta}\right), \quad x \to \infty ,$$

for some positive constants κ and ϵ.

- **Illustration:**
  - **If W is deterministic with mean a, the series diverges;**
  - **If W is exponential with mean a, the series converges.**

TIME SPACE CONNECTIVITY

- **Definition:** $\mathbb{G}_{\mathrm{SINR}}$ **is a.s. connected if for all** S **and** D **in** Φ **and all** $n \in \mathbb{Z}$**, there exists a path**
  - **from** (S, n)
  - **to** $\{(D, n+l)\}_{l \in \mathbb{N}}$**.**

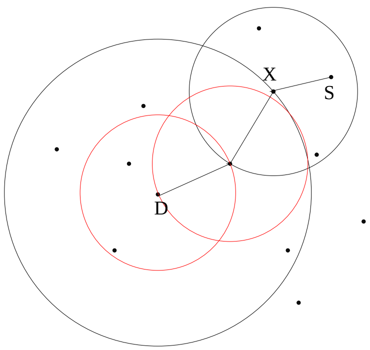


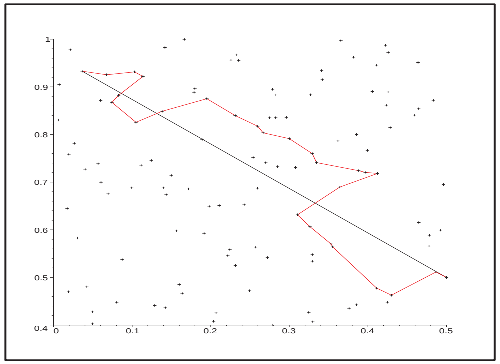
TIME SPACE CONNECTIVITY (*continued*)

- **Lemma Under either**
  - **(a) fast fading and F with unbounded support or**
  - **(b) W=0**

$\mathbb{G}_{\mathrm{SINR}}$ **is a.s. connected provided** $0 < p < 1$.

TIME SPACE CONNECTIVITY (*continued*)

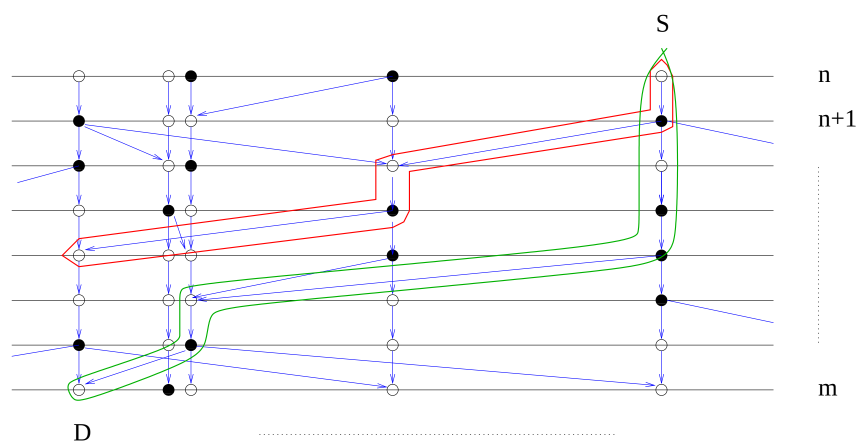




- **Idea of Proof**:
  - **Consider the static route where next hop from X is the point $\mathcal{A}(X)$ of the open ball $B_D(|X - D|)$ which is the closest to X.**
  - **Use the finiteness of local delay from X to $\mathcal{A}(X)$.**

FIRST PASSAGE PERCOLATION

- $|p^*(S, D, \Phi, n)|$: **number of time steps of the optimal (minimal number of time steps) path from** (S, n) **to** $\{(D, n+l)\}_{l \in \mathbb{N}}$.



A POSITIVE RESULT

- **Poisson network with an additional periodic infrastructure.**
- **Theorem In a $\frac{\mathbf{M}}{W+(\mathbf{M}+G_s)\,/\,\mathbf{M}}$ MANET model with fast Rayleigh fading, constant and positive thermal noise, power law attenuation,**

$$\kappa_{\mathrm{d}} = \lim_{|d-s|\to\infty} \overline{\frac{|p^*(s,d,\widetilde{\Phi})|}{|d-s|}}$$

exists, with κ_{d} a finite random variable which

– depends on the direction d of $\vec{s,d}$

– is measurable w.r.t. the random shift making the grid G_s stationary.

The convergence also holds in L_1.

A POSITIVE RESULT (*continued*)

- **Networking interpretation: Consider the time–space path(s) starting at time 0 from the point of Φ which is the closest to the origin and reaching the point of Φ which is the closest to $t = x \cdot$d$\in \mathbb{R}^2$ in the smallest possible number of time steps.**
- $\Delta^*(t)$**: number of time slots of this (these) path(s).**
- **The optimal path(s) has (have) a positive and finite asymptotic velocity v_d in direction d in the sense that**

$$\lim_{x \to \infty} \frac{x}{\Delta^*(x \cdot \mathrm{d})} = v_{\mathrm{d}}, \quad a.s.$$

IDEA OF PROOF

- **for** $t \in I\!\!R^2$**, let** $X(t)$ **be the point of** Φ **which is closest to** t**.**
- **For** $s, d \in I\!\!R^2$**, let** $p^*(s, d, \widetilde{\Phi}, k)$ **be a path of** $\mathbb{G}_{\mathrm{SINR}}$ **from** $(X(s), k)$ **to** $\{(X(d), l), l > k\}$**, with minimal number of time steps**
- **For all triples** s, v **and** d **in** $I\!\!R^2$**,**

$$|p^*(s, d, \widetilde{\Phi}, 0)| \leq |p^*(s, v, \widetilde{\Phi}, 0)| + |p^*(v, d, \widetilde{\Phi}, |p^*(s, v, \widetilde{\Phi}, 0)|)| \,.$$

- **Let**

$$\overline{|p^*(s, d, \widetilde{\Phi})|} = \mathbf{E}\left(|p^*(s, d, \widetilde{\Phi}, 0)| \;\mid \mathcal{G} \right)$$

with $\mathcal{G}$ **the** σ**-algebra generated by** Φ **(excluding the marks)**

$$\overline{|p^*(s, d, \widetilde{\Phi})|} \leq \overline{|p^*(s, v, \widetilde{\Phi})|} + \overline{|p^*(v, d, \widetilde{\Phi})|} \,.$$

IDEA OF PROOF (*continued*)

- **Existence and finiteness of the limit:**
- **From Kingman's subadditive ergodic theorem, and the property that for all points u, v of $\mathbb{R}^2$,**

$$\mathbf{E}\left[\sup_{u_1,v_1\in[u,v]} \overline{|p^*(u_1,v_1,\widetilde{\Phi})|}\right] < \infty\,,$$

where the supremum is taken over u_1, v_1 belonging to the interval $[u,v] \subset \mathbb{R}^2$.

- **This does not hold in the absence of periodic infrastructure (lack of integrability of local delays).**
- **Positiveness of the limit: large deviation bounds on the velocity based on the bounds on the degree and the Rayleigh assumptions.**

A NEGATIVE RESULT

- **Lemma** **In the $\frac{\mathbf{M}}{W+\mathbf{M}\,/\,\mathbf{M}}$ model with fast fading and power law attenuation, if W is positive and constant, for all s and d in $\mathbb{R}^2$,**

$$\mathbf{E}[\overline{|p^*(s,d,\widetilde{\Phi})|}] = \infty.$$

- **The proof is based on the fact that the ARR local delay of the first hop has infinite mean in this case.**

PALM SETTING

- **Theorem** **In the $\frac{\mathbf{G}}{W+\mathbf{M\,/\,G}}$ model with fast fading**

  1. **If the fading has unbounded support, then for all S, D,**

  $$\mathbf{E}^{S,D}[|p^*(S,D,\Phi)|] < \infty\,.$$

  2. **For fast Rayleigh fading and power law attenuation, if $W > w > 0$, then**

  $$\lim_{|S-D|\to\infty} \frac{\mathbf{E}^{S,D}[|p^*(S,D,\Phi)|]}{|S-D|} = \infty\,.$$

MAIN OBSERVATIONS ON THIS ROUTING PROBLEM

For Poisson Aloha networks with fast Rayleigh fading,

- **Local delays are finite at every point;**
- **The positiveness of the velocity of a priority packet on a route is not granted;**
- **For Rayleigh fading an positive thermal noise there exists no opportunistic routing algorithm with a positive asymptotic velocity.**
- **Phase transitions on velocity are possible.**

The increase of diversity and in particular the addition of a periodic infrastructure is enough to take care of positiveness issue.

CONCLUSIONS

– **SG allows one to compute spatial averages**

– **Interference can be represented by shot noise fields**

– **Basic MAC and routing protocols can be modeled using new SG objects based on SINR**

– **SG might in the long run play for wireless networks a role similar to that of QT in wired networks: compute macroscopic space/time averages, tune engineering parameters for stability, optimality, etc.**