2D polynomial approach to stability of platoons of vehicles

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Contribution

We formulate the problem of stability testing and stabilization of an infinite platoon of vehicles within the 2D polynomial framework, that is, the dynamics of the problem is described using a **fraction of two bivariate polynomials**.

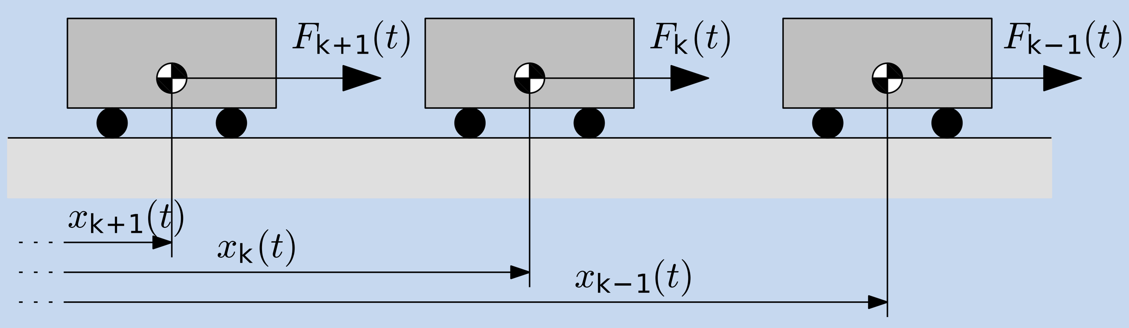
$$G(s,z) = \frac{b(s,z)}{a(s,z)}$$

Analysing the two-variate denominator polynomial $a(s,z)$ we can make conclusions about stability of a given feedback control configurations.

For feedback configurations that admit a stabilizing solution, recent computational tools for positive polynomials can be used for stabilization.

Model

(Doubly) infinite platoon of vehicles indexed with $k = \{\ldots,-1,0,1,2,\ldots\}$. The dynamics of each vehicle is described by a simple double integrator. Positions denoted as $x_k(t)$ and velocities as $v_k(t)$.



Bilateral z-transform is used in combination with Laplace tranform to get a transfer function relating the force $F_k(t)$ and distance to the vehicle ahead and self-velocity.

2D polynomial stability

Spatially distributed system described by the transfer function $G(s,z) = \frac{b(s,z)}{a(s,z)}$ with the two polynomials free of a common factor is BIBO stable **if** $a(s,z) \neq 0$ for all s and z such that $|z| = 1$ and $\Re(s) \geq 0$. Apply this test to the closed-loop characteristic polynomial

$$c(s,z) = a(s,z)p(s,z) + b(s,z)q(s,z)$$

Open Questions

The use of the bilateral z-tranform assumes a doubly infinite string of vehicles. The **absence of a leading vehicle** may be misleading. Modifying the results for use with a unilateral z-transform is the subject of our current research.

Some references

[Jovanović and Bamieh(2005)] On the ill-posedness of certain vehicular platoon control problems. *Automatic Control, IEEE Transactions on*, 50(9), 1307–1321.

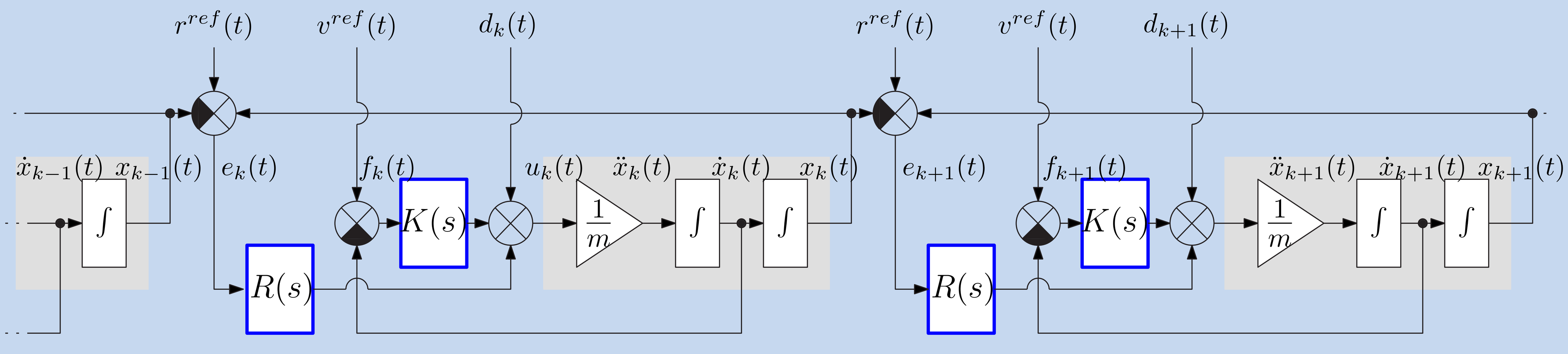
[Levine and Athans(1966)] On the optimal error regulation of a string of moving vehicles. *Automatic Control, IEEE Transactions on*, 11(3), 355–361.

[Melzer and Kuo(1971)] Optimal regulation of systems described by a countably infinite number of objects. *Automatica*, 7(3), 359–366.

[Peppard(1974)] String stability of relative-motion PID vehicle control systems. *Automatic Control, IEEE Transactions on*, 19(5), 579–581.

[Seiler et al.(2004)] Disturbance propagation in vehicle strings. *Automatic Control, IEEE Transactions on*, 49(10), 1835–1842.

Relative distance to the vehicle ahead and self-velocity available



With the distance to the vehicle ahead measured only, the transfer function is

$$G(s,z) = \frac{z^{-1}-1}{ms^2}$$

and the proportional controller is

$$C(s,z) = R$$

which makes the closed-loop polynomial

$$\boxed{c(s,z) = ms^2 + R\left(z^{-1}-1\right)}$$

With the the measurement of the absolute velocity

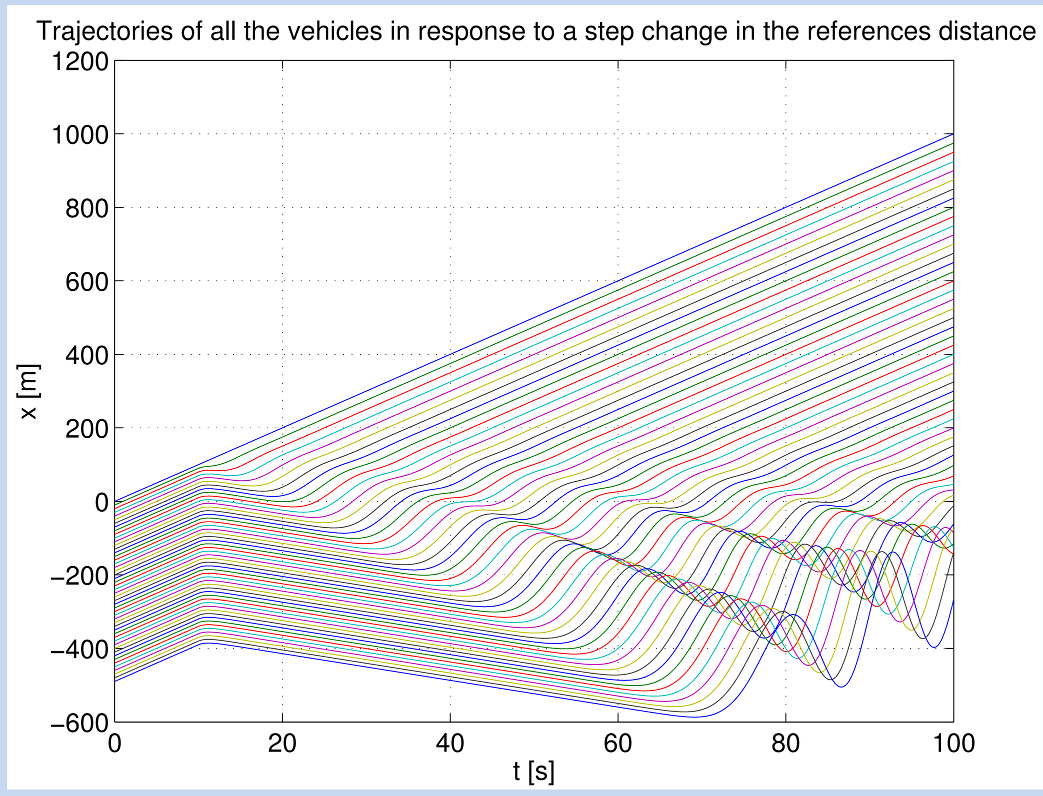
$$G(s,z) = \underbrace{\begin{bmatrix} z^{-1}-1 \\ s \end{bmatrix}}_{B(s,z)} \underbrace{(ms^2)^{-1}}_{(a(s,z))^{-1}}$$

and proportional 2-input controller is

$$C(s,z) = \begin{bmatrix} R & K \end{bmatrix}$$

which yields the closed-loop polynomial

$$\boxed{c(s,z) = ms^2 + Ks + R\left(z^{-1}-1\right)}$$



No way to make the closed-loop characteristic polynomial stable by R and K (setting K low will achieve boundary of instability).

Absolute position available

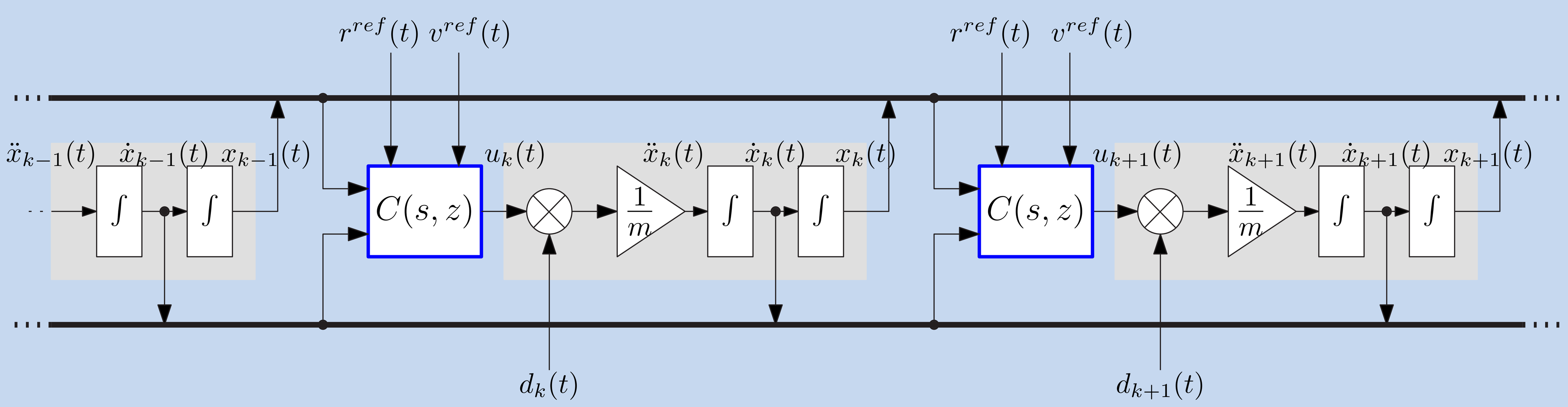
$$G(s,z) = \begin{bmatrix} z^{-1}-1 \\ s \\ 1 \end{bmatrix} \frac{1}{ms^2}$$

$$C(s,z) = \begin{bmatrix} R & K & P \end{bmatrix}$$

$$\boxed{c(s,z) = ms^2 + Ks + R\left(z^{-1}-1+P\right)}$$

Closed-loop stability can be achieved.

Global information available



All the measurements on the bus

$$C(s,z) = (p(s,z))^{-1} \underbrace{\begin{bmatrix} q_r(s,z) & q_v(s,z) \end{bmatrix}}_{Q(s,z)}$$

and the closed-loop polynomial is

$$\begin{aligned} c(s,z) &= p(s,z)a(s,z) + Q(s,z)B(s,z) \\ &= p(s,z)ms^2 + q_r(s,z)(z^{-1}-1) + q_v(s,z)s \end{aligned}$$

For a particular choice

$$p = (z^{-1}-1),\ q_r = R,\ q_v = K(z^{-1}-1)$$

the controller **spatially IIR**

$$\boxed{u_k(t) = u_{k-1}(t) + Rr_k(t) + Kv_k(t)}$$

and the closed-loop polynomial is

$$c(s,z) = (z^{-1}-1)ms^2 + R(z^{-1}-1) + K(z^{-1}-1)s$$

But $(z^{-1}-1)$ is not cancelled in all the closed-loop transfer functions ($r_{ref} \to u$). Nonessential singularity of the second kind appears. No way to BIBO stabilize with relative measurements?

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