Source Coding with Common Reconstruction and Action-dependent Side Information

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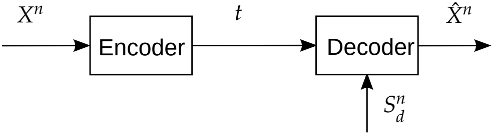
FeedNetback Junior Workshop, 16 September 2010

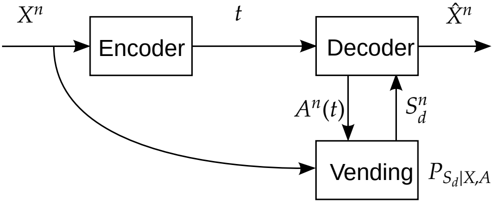
Outline

Related Work: Source Coding with Side Information

- Source coding with side information at the decoder [Wyner and Ziv '76]



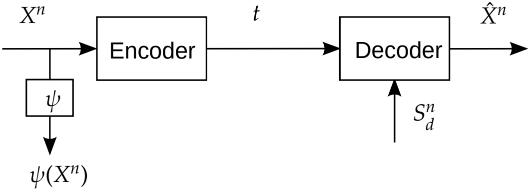
- Source coding with *action-dependent side information* "Vending machine" [Weissman and Permuter '09]



$\Rightarrow$ Decoder can adjust the quality of SI.

Related Work: Source Coding with Side Information

- Lossy source coding $\Rightarrow$ distortion constraint
- What if the encoder wants to know the decoder reconstruction as well? $\Rightarrow$ Common Reconstruction (CR) constraint [Steinberg '09]



$\Rightarrow$ $\lim_{n\to\infty} \Pr\big(\psi(X^n) \neq \hat{X}^n\big) = 0$

$\Rightarrow$ Medical consultation (MRI results) [Steinberg '09]

$\Rightarrow$ Both sender and receiver share common knowledge of receiver's reconstruction, i.e., $\hat{X}^n$.

Our Contribution

- Combining the action-dependent SI and CR constraint for the source coding problem.

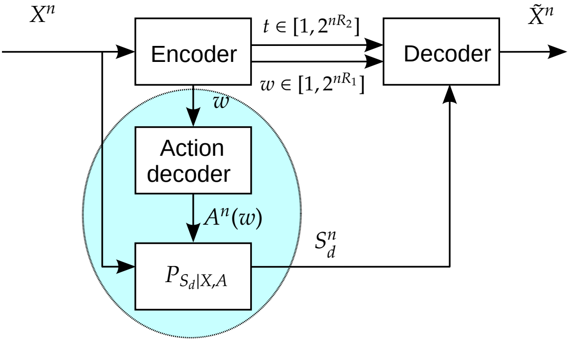


Figure: Action-dependent SI where action is chosen at the encoder and depends on a rate-limited link

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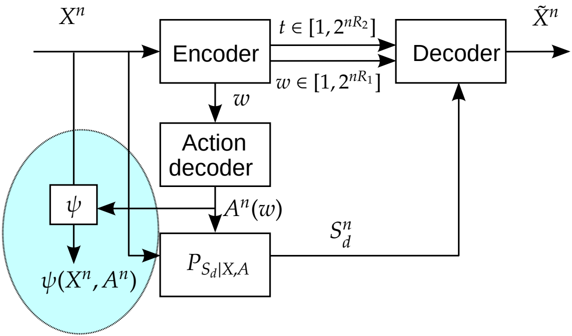
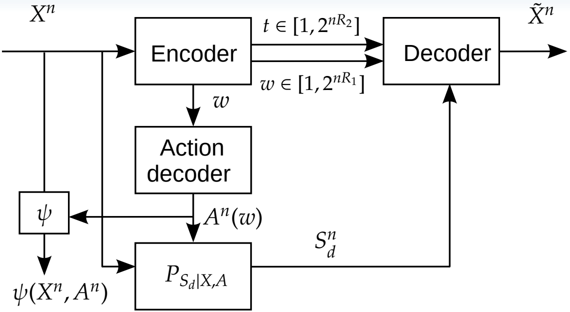


  - *Active* control: Action-dependent SI
  - *Passive* control: CR constraint
  - Potential application in networked control

Problem Formulation



- Encoder: $f^{(n)} : \mathcal{X}^n \to \mathcal{W}^{(n)} \times \mathcal{T}^{(n)}, |\mathcal{W}^{(n)}| = 2^{nR_1}, |\mathcal{T}^{(n)}| = 2^{nR_2}$
- Decoder: $g^{(n)} : \mathcal{W}^{(n)} \times \mathcal{T}^{(n)} \times \mathcal{S}_d^n \to \tilde{\mathcal{X}}^n$
- CR mapping: $\psi^{(n)} : \mathcal{X}^n \times \mathcal{A}^n \to \hat{\mathcal{X}}^n$, $\hat{\mathcal{X}}^n = \tilde{\mathcal{X}}^n$

Goal: To find the rate region subject to

$$E[d(X^n, \tilde{X}^n)] \le D, \; E[\Lambda(A^n)] \le C, \; \lim_{n\to\infty} \Pr\left(\psi^{(n)}(X^n, A^n) \ne \tilde{X}^n\right) = 0.$$

Rate Region

Definition: Rate Region

Let $\mathcal{P}(D, C)$ denote the set of all joint pmfs p which have the form

$$P_X(x)P_{A|X}(a|x)P_{S_d|X,A}(s_d|x,a)P_{\hat{X}|X,A}(\hat{x}|x,a),$$

and satisfy

$$E\Big[d\Big(X,\hat{X}\Big)\Big] \leq D, \text{ and } E[\Lambda(A)] \leq C.$$

For each $p \in \mathcal{P}(D, C)$, define

$$\mathcal{R}^*(D,C,p) \triangleq \Big\{(R_1,R_2) : R_1, R_2 \geq 0, R_1 + R_2 \geq I(X;\hat{X},A) - I(\hat{X};S_d|A)\Big\},$$

and

$$\mathcal{R}^*(D,C) \triangleq \bigcup_{p\in\mathcal{P}(D,C)} \mathcal{R}^*(D,C,p).$$

Rate Region

Theorem: Optimal Rate Region

The optimal rate region containing all achievable rate pairs for the memoryless source with action-dependent side information available at the decoder is given by

$$\mathcal{R}(D,C) = \mathcal{R}^*(D,C).$$

Remark:

$\Rightarrow$ No S_d in the distortion constraint $E\left[d\left(X,\hat{X}\right)\right] \leq D$ (different from that in Wyner-Ziv!)

Sum-rate Distortion and Cost Function

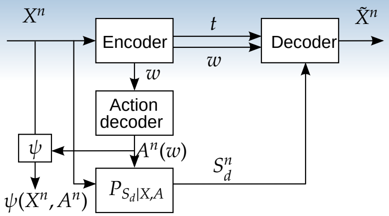
Corollary: Sum-rate distortion and cost function

The sum-rate distortion and cost function for the memoryless source with CR and action-dependent SI available at the decoder is given by

$$\begin{aligned} R_{ac,cr}(D,C) &= \min_{p \in \mathcal{P}(D,C)} [I(X;\hat{X},A) - I(\hat{X};S_d|A)] \\ &\stackrel{(*)}{=} \min_{p \in \mathcal{P}(D,C)} [I(X;A) + I(\hat{X};X|A,S_d)], \end{aligned}$$

$(*)$ follows from the Markov chain $\hat{X} - (X,A) - S_d$.

Outline of the Proof



The proof follows from arguments in [Wyner and Ziv '76], [Weissman and Permuter '09], and [Steinberg '09].

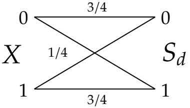
- Achievability: using a random coding argument
  - **Codebook generation**: $a^n(w) \sim P_A$, and $\hat{x}^n(t, v, w) \sim P_{\hat{X}|A}$
  - **Encoding**: given the source, find (t, v, w) s.t. $\hat{x}^n, x^n$, and a^n are jointly typical, then transmit t, w to the decoder and w to the action decoder. Also, put out $\hat{x}^n(t, v, w)$ as CR.
  - **Decoding**: put out $\tilde{x}^n = \hat{x}^n$ which is jointly typical with a^n and s_d^n

  $\Rightarrow$ Typicality guarantees the distortion, cost and CR constraints.

- Converse: standard information-theoretic argument

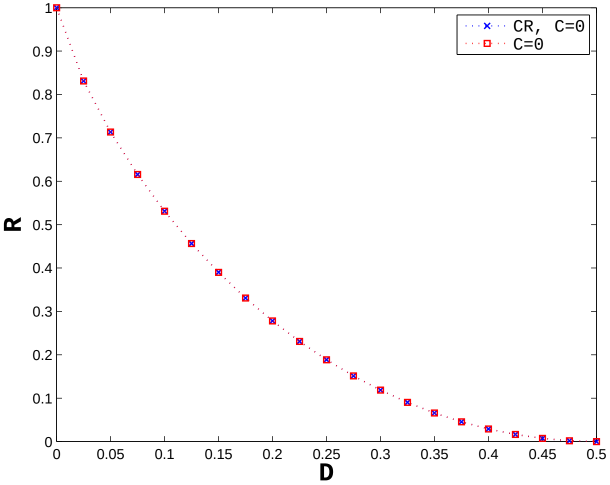
Example: To observe, or not to observe SI

- Comparing $R_{ac,cr}(D,C)$ to $R_{ac}(D,C)$ (without CR constraint)
  - Consider the binary sets $\mathcal{X} = \hat{\mathcal{X}} = \mathcal{S}_d = \mathcal{A} = \{0,1\}$
    - $X \sim$ Bern(1/2)
    - $A = 1 \Rightarrow$ observe S_d
    - $A = 0 \Rightarrow$ not observing it



  - Assume that an observation has a unit cost, i.e., $\Lambda(A) = A$ and $E[\Lambda(A)] = P_A(1) = C$
  - Consider the Hamming distance as a distortion measure

Rate-Distortion Curves



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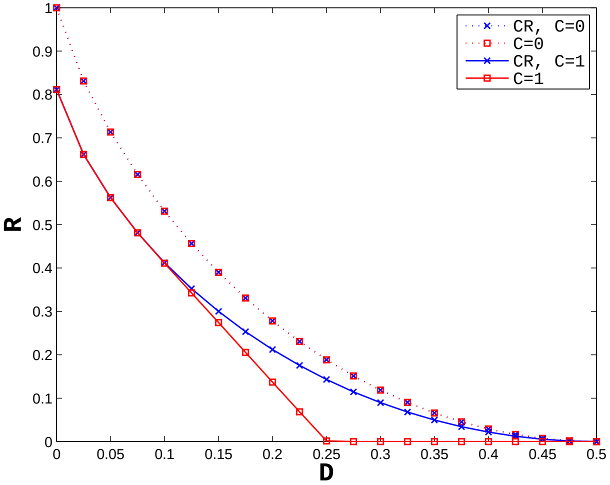

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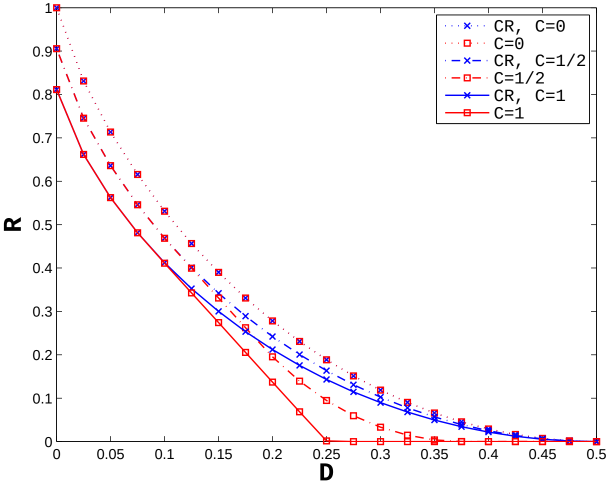

- Performance loss due to CR constraint

Summary

- Source coding problem with action-dependent SI and CR (extension of Wyner-Ziv)
- The rate region is characterized (depending only on the sum rate constraint)
- Potential application in networked control

Thank you for your attention!

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References

K. Kittichokechai, T.J. Oechtering and M. Skoglund
Source coding with common reconstruction and action-dependent side information.
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