A Nyquist criterion for synchronization in networks of heterogeneous linear systems

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Outline

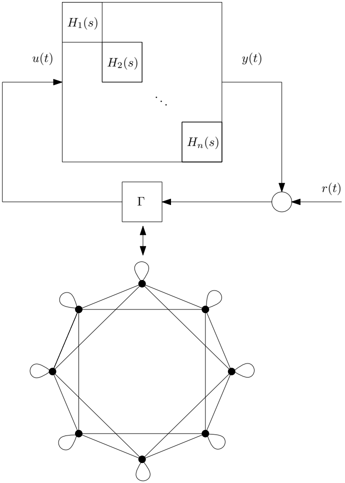
- Problem formulation
- Main result
- Application: double integrators synchronization

Problem formulation

Network of n interconnected heterogeneous agents

Problem formulation

Network of *n* interconnected heterogeneous agents



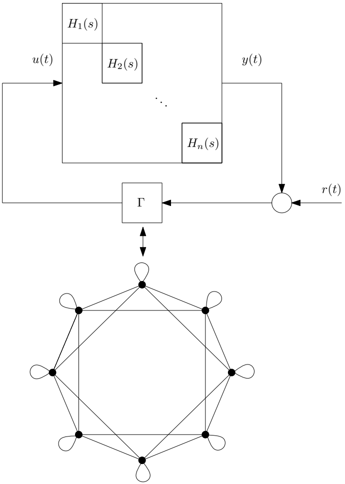
Problem formulation

Network of n interconnected **heterogeneous** agents

$\longrightarrow$ *Heterogeneous*

SISO LTI subsystems

$y_k(t) = H_k(s)u_k(t)$



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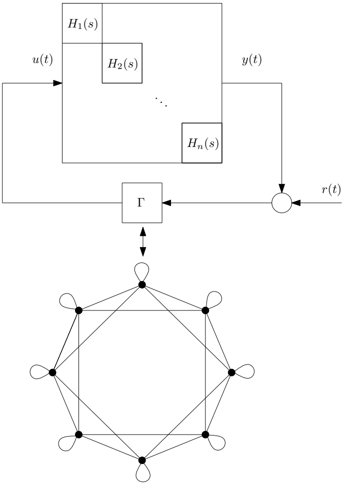


Problem formulation

Network of n **interconnected** heterogeneous agents

$\longrightarrow$ Heterogeneous

SISO LTI subsystems

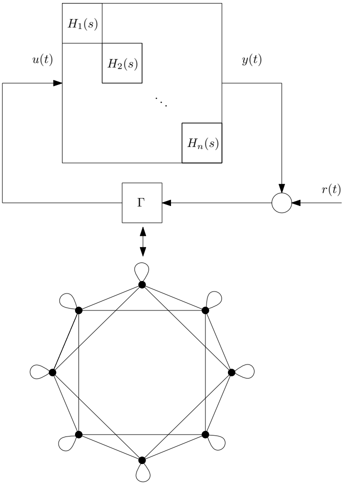
$y_k(t) = H_k(s) u_k(t)$

$\longrightarrow$ *Interconnected*

$u(t) = \Gamma y(t) + r(t)$

Γ is a normal Laplacian of $\mathcal{G}$

$\Gamma \mathbf{1} = 0$ and $\dim \ker \Gamma = 1$



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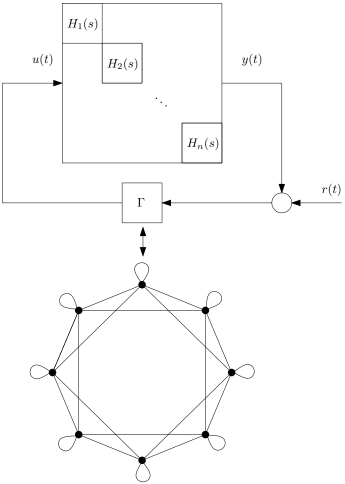

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Problem formulation

Goal

We want to give sufficient conditions on Γ and $H_k(s)$ in order to **synchronize** the network

$$\|y_i(t) - y_j(t)\| \xrightarrow{t \to \infty} 0.$$

Problem formulation

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We want to give sufficient conditions on Γ and $H_k(s)$ in order to **synchronize** the network

$$\|y_i(t) - y_j(t)\| \xrightarrow{t \to \infty} 0.$$

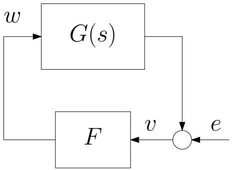
Remark: once the system is synchronized, the input signal r can be easily used in order to perform higher level tasks, e.g. formation control.

Main result - Inspiration

Basis: Integral Quadratic Constraint (IQC) theorem.

Consider the LTI operator $G(s)$ in feedback with operator F

$$\begin{cases} v = Gw + e \\ w = F(v) \end{cases}$$



Sufficient condition for I/O stability is the existence of set of *multipliers* such that, in a suitable Hilbert space, $G(s)$ and F are "separated" by the set of multipliers.

Main result - Inspiration (2)

Related results: Small Gain Theorem, Passivity Theorem.

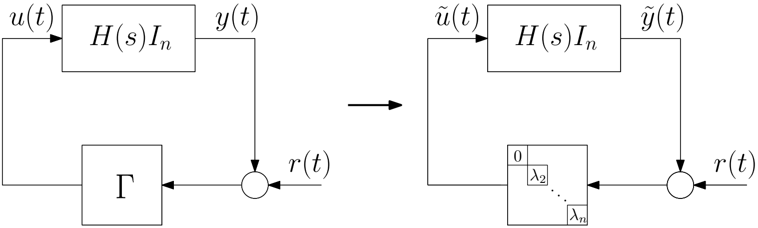
References

- A. Megretski and A. Rantzer, *System analysis via integral quadratic constraints*, IEEE TAC, 1997
- C.-Y. Kao, U.T. Jönsson, and H. Fujioka, *Characterization of robust stability of a class of interconnected systems*, Automatica, 2010 (to appear)

Main result - Prior Work

J.A. Fax and R.M. Murray, *Information flow and cooperative control of vehicle formations*, TAC 2004

In the homogeneous case ($H_k(s) = H(s)$) synchronization takes place if **Nyquist criterion** holds for $-\frac{1}{\lambda_k}$, for any nonzero eigenvalue λ_k of Γ.



Main result

Structure of the subsystems

$$H_k(s) = N_0(s) + N_k(s)$$

where $N_0(s)$ is the "nominal" plant, $N_k(s)$ is a perturbation.

Goal

Take $\alpha > 0$. We want to find sufficient conditions for synchronization of "destabilized outputs" $e^{\alpha t}y(t)$ under the condition that $e^{\alpha t}r(t), e^{\alpha t}\dot{r}(t) \in \mathbf{L}_2[0, \infty)$.

Main result (2)

Conditions on the subsystems

i) $W_0(s-\alpha) = \frac{N_0(s-\alpha)}{1-N_0(s-\alpha)\lambda_k}$ is stable $\forall\, \lambda_k \neq 0$

ii) $1 - N_0(s-\alpha)\lambda_k$ is nonsingular on the imaginary axis

iii) $\frac{N_k(s-\alpha)}{N_0(s-\alpha)}$ are stable $\forall\, k$

Remark: $i) - ii)$ is the condition of Fax and Murray on the nominal system

Main result (3)

Condition on the interconnection (IQC-like)

$$\mathcal{N}\left[H_0, \ldots, H_n\right](j\omega - \alpha) \cap \Omega = \emptyset, \forall\, \omega \in \mathbf{R} \cup \{\infty\}$$

where

$$\mathcal{N} = \mathrm{co}\{\left(\mathrm{Re} H_k,\, \mathrm{Im} H_k,\, |H_k|^2\right), \forall\, k\}$$

$$\Omega = (0,\, 0,\, \mathbb{R}^+) + \mathrm{co}\left\{\left(\mathrm{Re}\frac{1}{\lambda_k},\, \mathrm{Im}\frac{1}{\lambda_k},\, \frac{1}{|\lambda_k|^2}\right), \forall\, k\right\}$$

Main result (4)

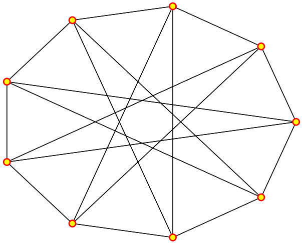
Remarks:

- similar results can be found switching the role of the subsystems an of Γ and/or projecting the obtained regions over the complex plane.
- analogous criteria exist for discrete time case

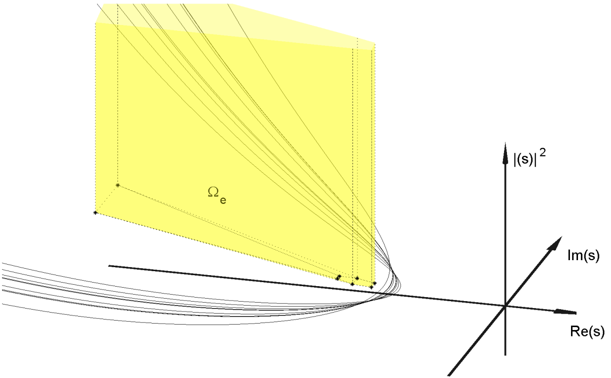
Synchronization of double integrators: a toy example

Consider a set of agents on the plane. We assume that we can decouple the trajectories on the x-axis and on the y-axis.
We model each agent as a double integrator plus a common control

$$\begin{cases} \dot{x}_k(t) = \begin{bmatrix} 0 & q_k \\ 0 & 0 \end{bmatrix} x_k(t) + \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} u_k(t) \\ y_k(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x_k(t) \\ u_k(t) = \sum_{j \in \mathcal{N}_k} \Gamma_{kj} y_j(t) \end{cases}$$

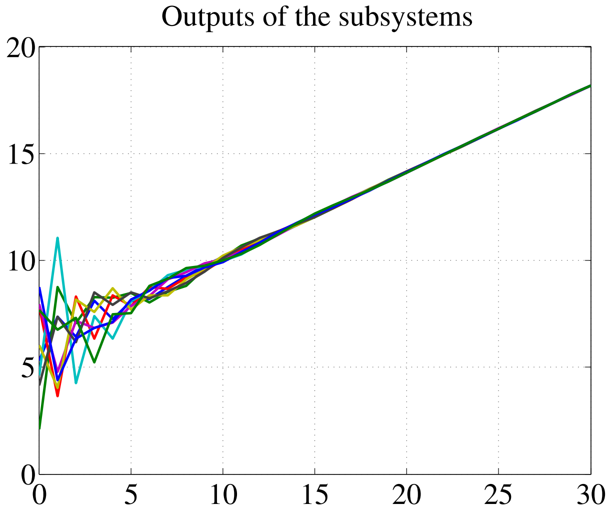


Synchronization of double integrators: a toy example (2)



The Nyquist criterion is satisfied.

Synchronization of double integrators: a toy example (2)



Trajectory on the x-axis.

Thanks!

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