Lyapunov-Krasovskii functionals for the study of stability and stabilisation of time-delay systems with application to networked control systems

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Introduction

Stability of time-delay systems

- Lyapunov functions for systems without delays.
- Time-delay systems Lyapunov-Razumikhin (LRF) and Lyapunov-Krasovskii functionals (LKF).
- Results for continuous and discrete systems.

Review work

The LKF approach

Goals

- Analyze stability of linear time delay systems [1].
- Robust stability analysis [2].
- Robust controller designs [3].
- Optimal controller designs [4].
- Mixed $H_2/H_\infty$ controllers [4].

Related publications


Non reliable communication channel

- Sampling.
- Randomly time-varying delays.
- Packages dropouts.
- Other: quantization, energy aware, etc.

Possible consequences

- Degradation of the control performance
- "Expensive" control
- Unstable behaviours
We work with **Linear Time Invariant Systems (LTI)** with differentiated uncertainties, disturbances and non-ideal networked links in sensor-to-controller and controller-to-actuator paths.

\[
\dot{x}(t) = Ax(t) + Bu(t) + B_\omega \omega(t), \\
z(t) = Cx(t) + Du(t), \\
x(t_0) = x_0,
\]

- Sensor nodes sample data in a time-driven manner at time instants \( t = j_k h \) such that \( \{j_1, j_2, j_3, \ldots\} \subseteq \{1, 2, 3, \ldots\} \) and \( j_k < j_{k+1} \).
- \( t \in [t_k, t_{k+1}) \) time intervals with constant control input.
SYSTEM UNCERTAINTIES

**Parametric uncertainties**

\[ [A, B] \rightarrow [A^* + \Delta A(t), B^* + \Delta B(t)] \]

\[ \Delta A(t) = G_1 F_1(t) E_1, \]
\[ \Delta B(t) = G_2 F_2(t) E_2 \]

**Polytopic uncertainties**

\[ \Omega = [A \ B]/\Omega \in \text{conv}\{\Omega_j, j = 1, \ldots, N\}, \]
\[ \Omega_j = [A^{(j)} \ B^{(j)}]. \Omega = \sum_{i=1}^{12} \sum_{j=1}^{N} f_j \Omega_j, \]
\[ 0 \leq f_j \leq 1, \sum_{j=1}^{N} f_j = 1. \]

Close loop NCS model \((u = Kx)\)

\[ \dot{x}(t) = Ax(t) + BKx(t - \tau(t)) + B_\omega \omega(t), \quad \forall t \in [t_k, t_{k+1}), \]
\[ z(t) = Cx(t) + DKx(t - \tau(t)), \quad \forall t \in [t_k, t_{k+1}), \]
\[ x(t) = \phi(t), \quad t \in [t_0 - \tau_M, t_0], \]

where \(\tau(t) = t - t_k + \tau_{sc}(k) + \tau_{ca}(k)\)

Sampling period \(h\), Round-Trip delay \(\tau_{RT}\) and consecutive package losses \(N_p\) bounded imply that \(\tau(t)\) is bounded:

\[ \tau_{RT} \leq \tau(t) \leq \tau_{RT} + (h + 1)N_p \]
General procedure

Notation

- $\zeta$ Set of network parameters: $\tau_M, \tau_m, N_p, h$
- $\xi^T(t) = [x^T(t), x^T(t - \tau(t)), ...]$
- $V(t, \zeta)$ Candidate to be a Lyapunov-Krasovskii functional
- $J = \int_{t_0}^{\infty} [\xi^T(t)\Phi(K)\xi(t)]dt$ Cost function.

LINEAR MATRIX INEQUALITIES FOR DIFFERENT CONTROL PROBLEMS

System stability

$$\frac{d}{dt} V(t, \zeta) = \epsilon^T(t)\Xi(K, \zeta)\epsilon(t), \quad \Xi(K, \zeta) < 0$$

Constraint on the system $L_2$ gain

$$\frac{d}{dt} V(t, \zeta) = \epsilon^T(t)\Xi\epsilon(t) + z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t), \quad \Xi(K, \zeta) < 0$$

Optimal control

$$\min_K \alpha,$$

subject to $\alpha\Xi(K, \zeta) < -\Phi(K)$

$$\alpha > 0, \quad \alpha \in \mathbb{R}$$
### Technical arguments

1. Schur complement  
2. S-procedure  
3. Finsler’s lemma  
4. Leibnitz-Newton equation  
5. Jensen’s inequality  
6. Moon’s inequality  
7. Slack matrices  
8. Polytopic descriptions  
9. Cone complementary algorithm

Control problems can be written in terms of **Linear Matrix Inequalities**
Application of the LK Approach to design $H_2/H_\infty$ controllers

Vehicle tracking system:

\[
e(t) = p_1(t) - p_2(t) - l_r,
\]
\[
\dot{e}(t) = v_1(t) - v_2(t) = y(t),
\]
\[
\dot{y}(t) = a_1(t) - a_2(t).
\]

Forces equilibrium:

\[
F_i(t) - F_{a,i}(t) - F_{r,i}(t) = m_i a_i(t), \quad i = 1, 2.
\]

Aerodynamical drag: \( F_a(t) = \frac{1}{2} c_a A_T \rho \text{air} v^2(t) \),
Rolling friction: \( F_r(t) = c_r m g \cos(\alpha(t)) \).

System’s linearization

\[
\frac{d}{dt} \begin{pmatrix} \int e(t) \\ e(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -c_2 c_1 \end{pmatrix} \begin{pmatrix} \int e(t) \\ e(t) \\ y(t) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -c_3 \end{pmatrix} F_2(t) + \begin{pmatrix} 0 \\ 0 \\ c_3 \end{pmatrix} F_1(t)
\]

Control Law:

\[
F_2(t) = K \begin{pmatrix} \int e(t - \tau(t)) \\ e(t - \tau(t)) \\ y(t - \tau(t)) \end{pmatrix}
\]

Parameters

- \( c_a, c_r \): aerodynamic and tire-road drag coefficients, respectively.
- \( A_T \): vehicle’s aerodynamic cross-section.
- \( \rho \text{air} \): air density.
- \( m \): vehicle mass.
- \( g \): gravity constant.
- \( \alpha(t) \): road slope angle.
Local and Networked Control

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Summary

Lyapunov-Krasovskii functionals for the study of stability and stabilisation of time-delay systems with application to networked control systems
Response with network effects for various disturbance attenuation levels
Trading off performance vs. control effort
Conclusions

- Closing control loops over communication networks requires the participation of new analysis and design techniques.
- The LK approach makes possible the design of controller considering different specification.
- The problems are solved using LMI.
- Simulation examples shows the good performance of the controllers designed with this approach.
Thank you for your attention.