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Analytical Model of IEEE 802.15.4 for Multi-hop Networks

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Motivations

- Distributed wireless applications set new requirements and limitations on network protocols
  - High reliability, low latency, energy efficiency
- Multi-hop communication is instrumental for many application scenarios.
- Ongoing standardization for routing over low-power lossy networks (ROLL) by IETF.
- IEEE 802.15.4 standard for MAC and physical layer in WPAN.

→ Analytical study of the interaction between routing decision and MAC performances of IEEE 802.15.4 in multi-hop networks with ROLL specifications.

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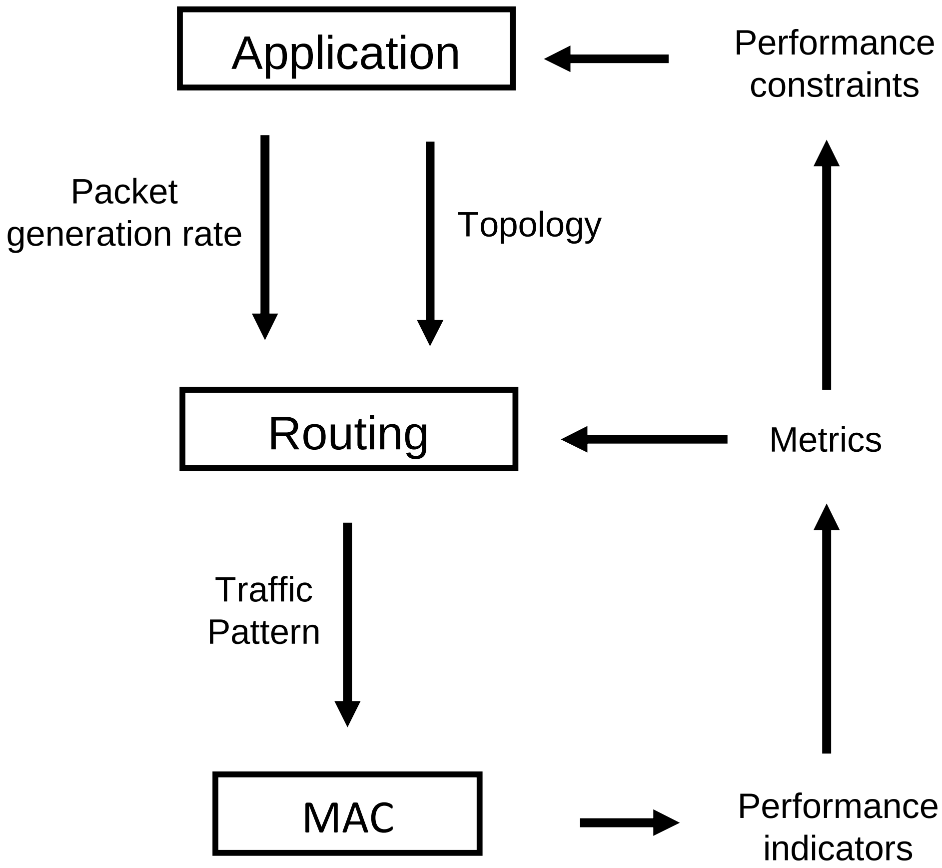
Protocol Interactions



- RPL - Routing over low power and lossy networks
  - DODAG (Destination-oriented Directed Acyclic Graph) structure
  - Multipoint-to-point communications
- Unslotted IEEE 802.15.4 MAC
  - Random access based on CSMA/CA with binary exponential backoff (BEB)

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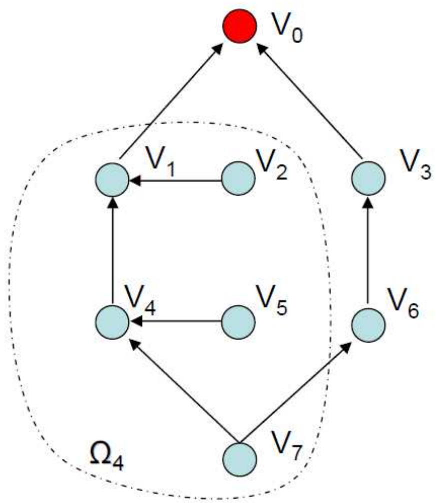
System Model

- Key aspects of multihop networks
  - Heterogeneous traffic condition
  - Limited carrier sensing range (hidden terminal problem)
- Topology
  - Ω_i: neighborhood set of node *i*
  - $\Omega_{j/i}$: hidden terminal set of link *(i,j)*
  - Γ_i: parent set of node *i*
  - Δ_i: children set of node *i*



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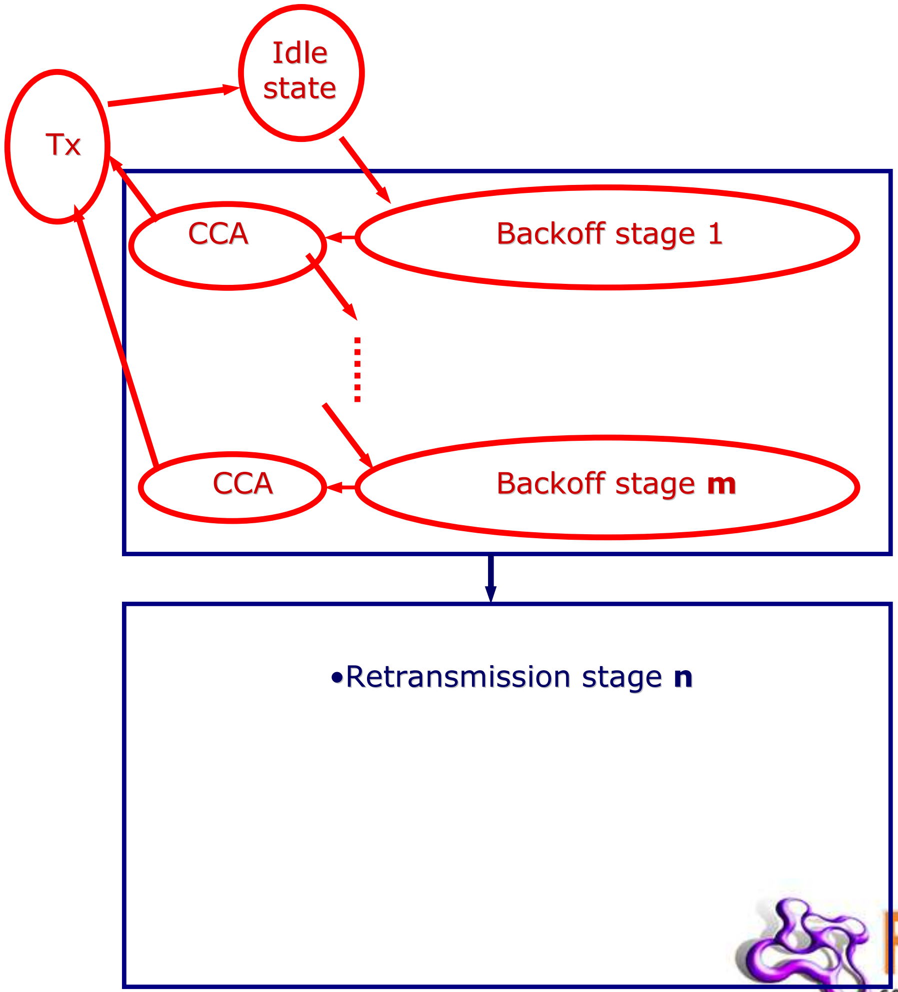
Unslotted IEEE 802.15.4 MAC



- Binary Exponential Backoff (BEB):

  - A transmitting node delays for a random number of backoff periods in [0 , 2^{m_0} - 1], where m_0 is the initial backoff exponent.
  - If clear channel assessments (CCA) is idle, the node starts the transmission and waits for an ACK.
  - If the channel is busy, the procedure is repeated increasing the backoff window until a maximum backoff exponent m_b.
  - After a maximum number of backoffs m the packet is discarded.
  - In case of collision the procedure is restarted and repeated until a retry limit n

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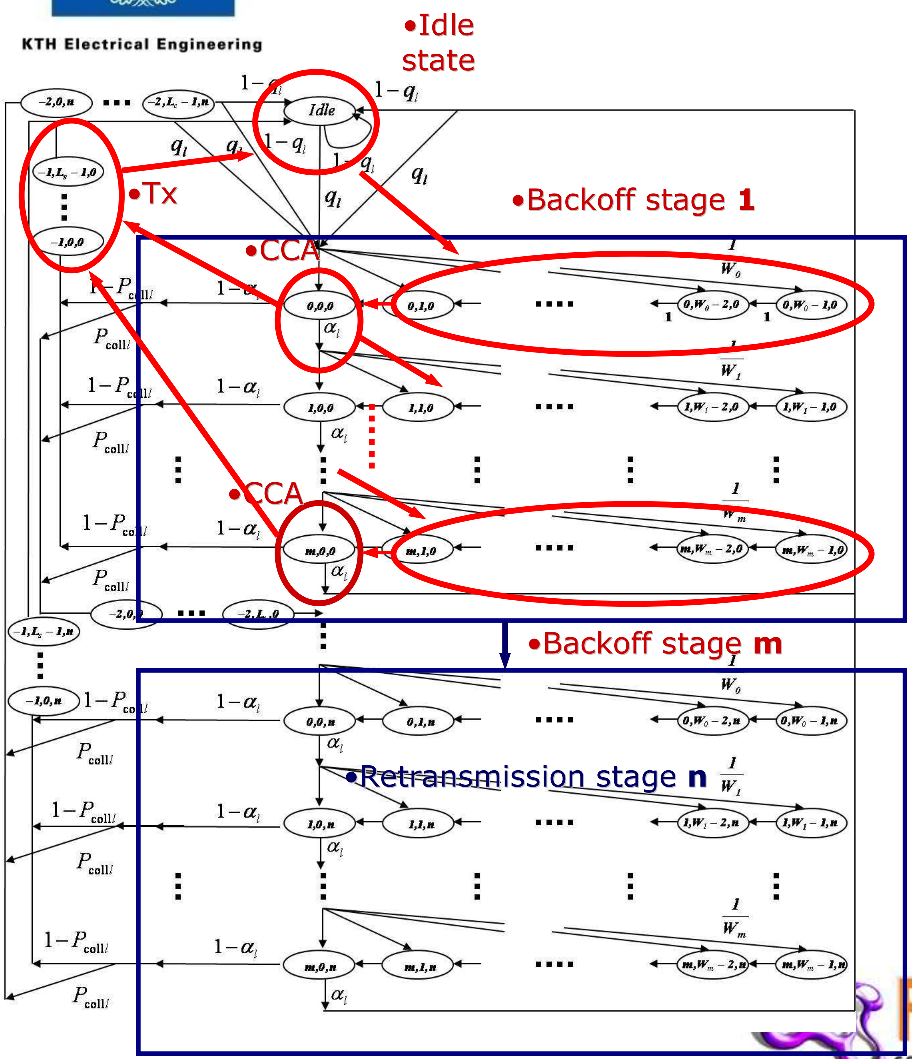
Markov Chain Model



- Three dimensional Markov chain (s,c,r)
  - s: backoff stage
  - c: state of backoff counter
  - r: state of retransmission counter
- Model characteristic parameter
  - $\lambda=1/q_0$: traffic condition
  - m_0, m, m_b, n: MAC parameters
- Computed model parameters
  - α_l: busy channel probability during CCA
  - $P_{coll,l}$: collision probability

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Model Equations

- Channel access probability

$$\tau_l = \left(\frac{1-\alpha_l^{m+1}}{1-\alpha_l}\right)\left(\frac{1-y_l^{n+1}}{1-y_l}\right) b_{0,0,0}^{(l)}$$

$$b_{0,0,0}^{(l)} = \begin{cases} \Big[\frac{1}{2}\left(\frac{1-(2\alpha_l)^{m+1}}{1-2\alpha_l}W_0 + \frac{1-\alpha_l^{m+1}}{1-\alpha_l}\right)\frac{1-y_l^{n+1}}{1-y_l} + (L_s(1-P_{\mathrm{coll},l}) + L_c P_{\mathrm{coll},l})(1-\alpha^{m+1})\frac{1-y_l^{n+1}}{1-y_l} \\ +\frac{1-q_l}{q_l}\left(\frac{\alpha_l^{m+1}(1-y_l^{n+1})}{1-y_l} + P_{\mathrm{coll},l}(1-\alpha_l^{m+1})y_l^n + (1-P_{\mathrm{coll},l})\frac{(1-\alpha_l^{m+1})(1-y_l^{n+1})}{1-y_l}\right)\Big]^{-1}, \\ \quad \text{if } m \le \bar{m} = m_b - m_0 \\ \\ \Big[\frac{1}{2}\left(\frac{1-(2\alpha_l)^{\bar{m}+1}}{1-2\alpha_l}W_0 + \frac{1-\alpha_l^{\bar{m}+1}}{1-\alpha_l} + (2^{m_b}+1)\alpha_l^{\bar{m}+1}\frac{1-\alpha_l^{m-\bar{m}}}{1-\alpha_l}\right)\frac{1-y_l^{n+1}}{1-y_l} \\ +(L_s(1-P_{\mathrm{coll},l}) + L_c P_{\mathrm{coll},l})(1-\alpha^{m+1})\frac{1-y_l^{n+1}}{1-y_l} \\ +\frac{1-q_l}{q_l}\left(\frac{\alpha_l^{m+1}(1-y_l^{n+1})}{1-y_l} + P_{\mathrm{coll},l}(1-\alpha_l^{m+1})y_l^n + (1-P_{\mathrm{coll},l})\frac{(1-\alpha_l^{m+1})(1-y_l^{n+1})}{1-y_l}\right)\Big]^{-1}, \\ \quad \text{otherwise,} \end{cases}$$

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Model Equations

- Busy channel probability

$$\alpha_{\text{pkt},l} = L \sum_{i=0}^{|\Omega_l|-2} \sum_{j=1}^{C_{l,i}} \prod_{k=1}^{i+1} \tau_{k_j} \left(1 - \prod_{k=1}^{i+1} \alpha_{k_j}\right) \prod_{h=i+2}^{|\Omega_l|} (1 - \tau_{h_j})$$

$$\alpha_{\text{ack},l} = L_{\text{ack}} \sum_{h \in \Omega_0 h \neq l} q_h R_h$$

- Collision probability

$$P_{\text{coll},l} = \Pr[\mathcal{A}_l] + (1 - \Pr(\mathcal{A}_l)) \Pr[\mathcal{B}_l]$$

$$\Pr[\mathcal{A}_l] = 1 - \prod_{k \in \Omega_0 k \neq l} (1 - \tau_k)$$

$$\Pr[\mathcal{B}_l] = 2L \sum_{i=0}^{|\Omega_{0 \setminus l}|-2} \sum_{j=1}^{C_{l,i}} \prod_{k=1}^{i+1} \tau_{k_j} \left(1 - \prod_{k=1}^{i+1} \alpha_{k_j}\right) \prod_{h=i+2}^{|\Omega_l|} (1 - \tau_{h_j})$$

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Routing Model

- Let $\lambda \in \Re^N$ be a vector of node packet generation rates.
- Let $\pi_{i,j}$ be the metric associated to link *l=(i,j).*

- The effect of the routing is described by a matrix *T* such that:

$$T_{i,j} = \Pr\left[\pi_{i,j} = \max_{V_h \in \Gamma_i} \pi_{i,h}\right]$$

$$\widetilde{T}_{i,j} = T_{i,j} R_l$$

- In stationary conditions, the vector of traffic rates can be described by a system of flow balance equations:

$$Q = Q\,\widetilde{\mathbf{T}} + \lambda \qquad\qquad Q \in \Re^N$$

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Performance Indicators

- Reliability
  - Homogeneous link quality

$$R_l = 1 - (P_{\mathrm{cf},l} + P_{\mathrm{cr},l})$$

$$P_{\mathrm{cf},l} = \frac{\alpha_l^{m+1}(1 - (P_{\mathrm{coll},l}(1 - \alpha_l^{m+1}))^{n+1})}{1 - P_{\mathrm{coll},l}(1 - \alpha_l^{m+1})}$$

$$P_{\mathrm{cr},l} = (P_{\mathrm{coll},l}(1 - \alpha_l^{m+1}))^{n+1}$$

- Delay
  - Measured for successfully received packets

$$\mathbb{E}[D_l] = \sum_{j=0}^{n} \Pr(\mathscr{C}_j|\mathscr{C})\,\mathbb{E}[D_{l,j}]$$

$$\Pr(\mathscr{C}_j|\mathscr{C}) = \frac{\left(1 - P_{\mathrm{coll},l}(1 - \alpha_l^{m+1})\right) P_{\mathrm{coll},l}^{j}(1 - \alpha_l^{m+1})^j}{1 - \left(P_{\mathrm{coll},l}(1 - \alpha_l^{m+1})\right)^{n+1}}$$

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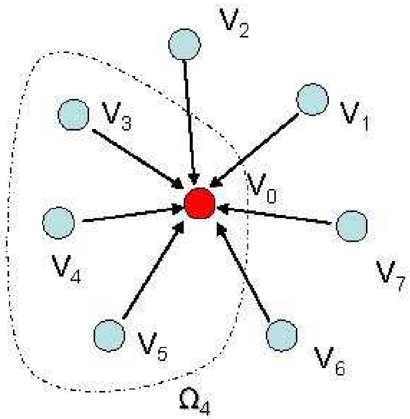
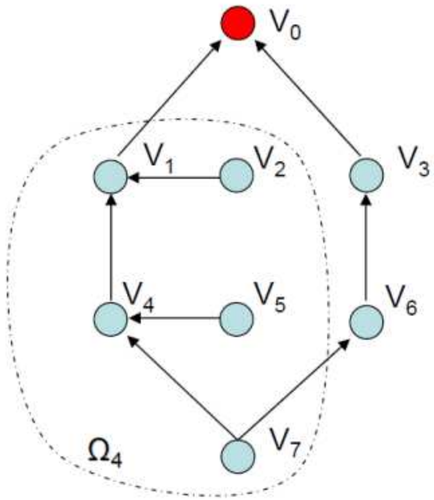
Simulation Setup

- Single-hop Analysis
  - Traffic condition
    - Homogeneous (λ_i=5 pkt/s i=1,…,N)
    - Heterogeneous (λ_4=20 pkt/s; λ_i=5 pkt/s)
  - Sensing range
    - Full sensing ($|\Omega_i|$=N i=1,…,N)
    - Reduced sensing ($|\Omega_i|$=3 i=1,…,N)



- Multi-hop Analysis
  - Path selection
    - Path 1 (V_7-V_4-V_1-V_0)
    - Path 2 (V_7-V_6-V_3-V_0)
  - Coupling
    - Coupled paths
    - Isolated paths



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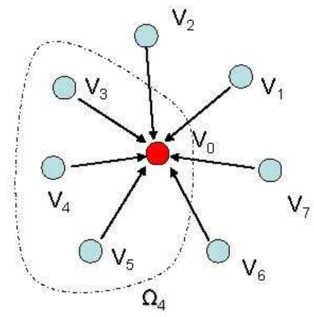
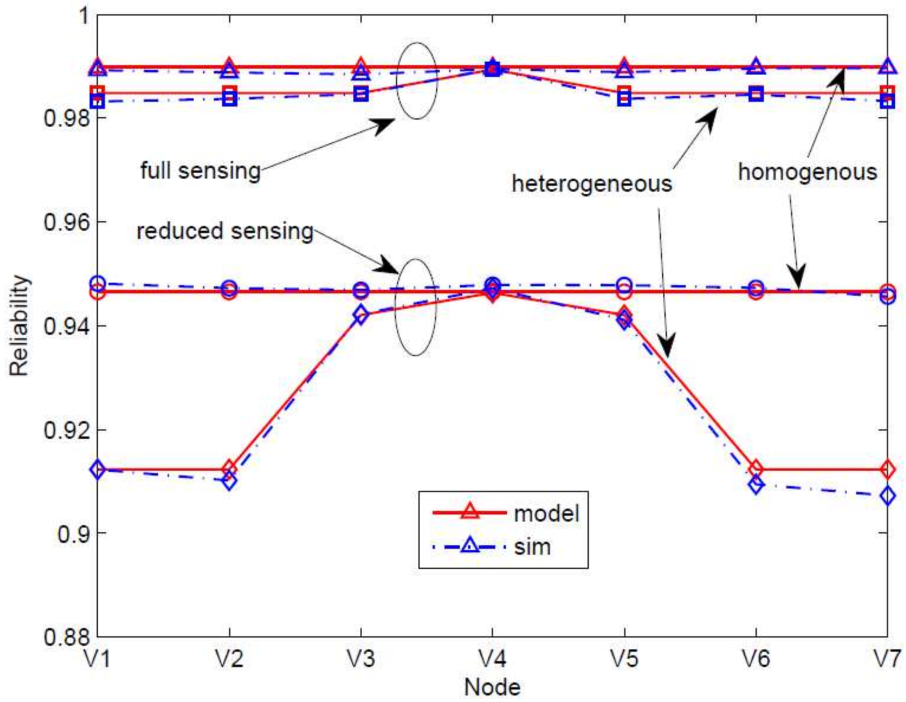
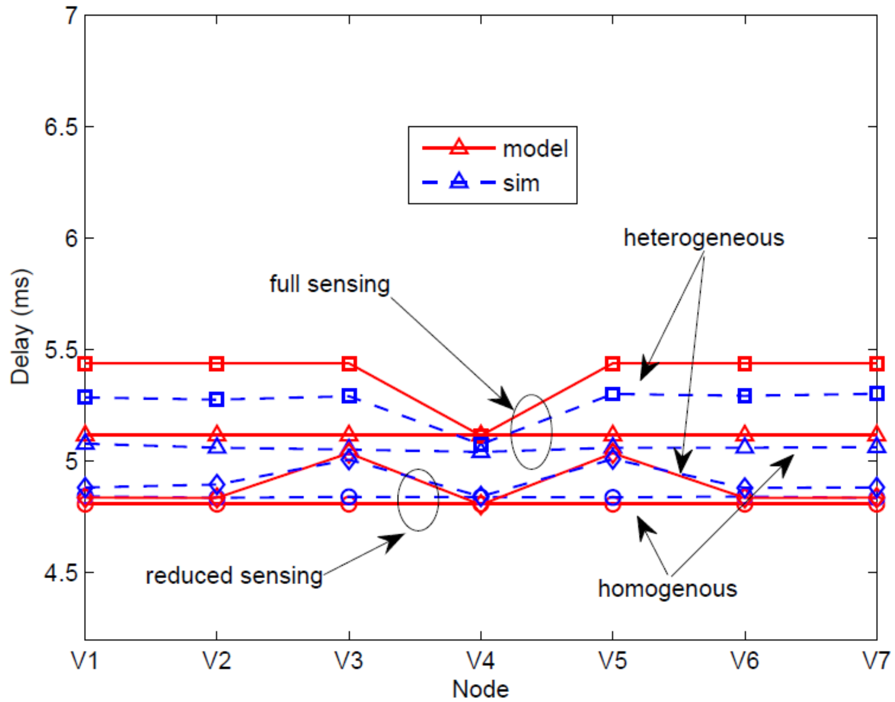
Single-hop Performance







Reliability increases when the number of contenders and hidden terminals decreases

Delay decreases when the number of contenders decreases

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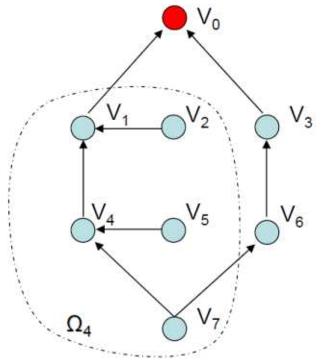
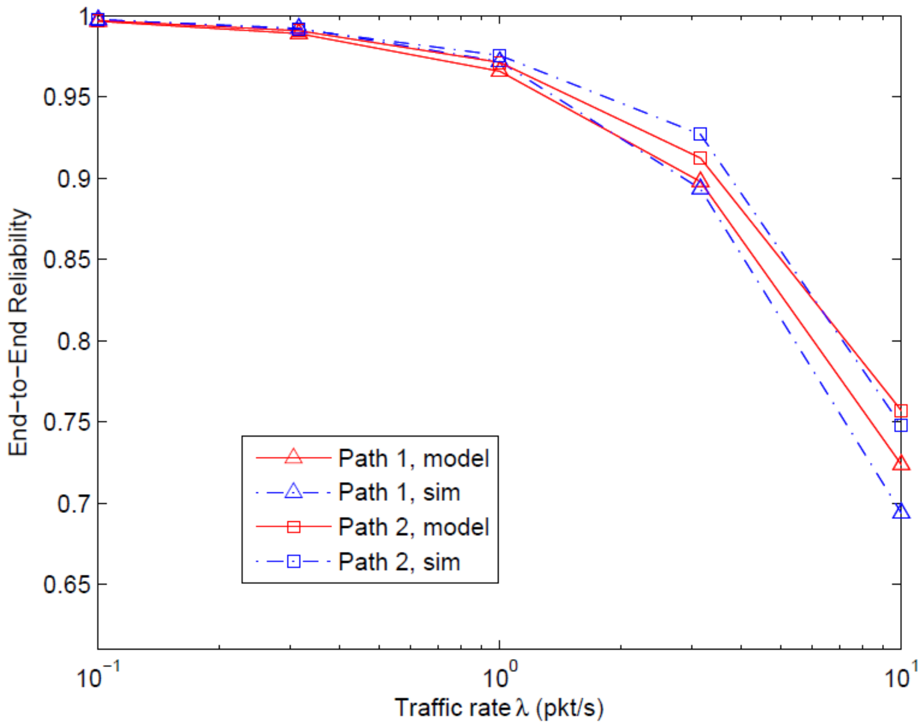
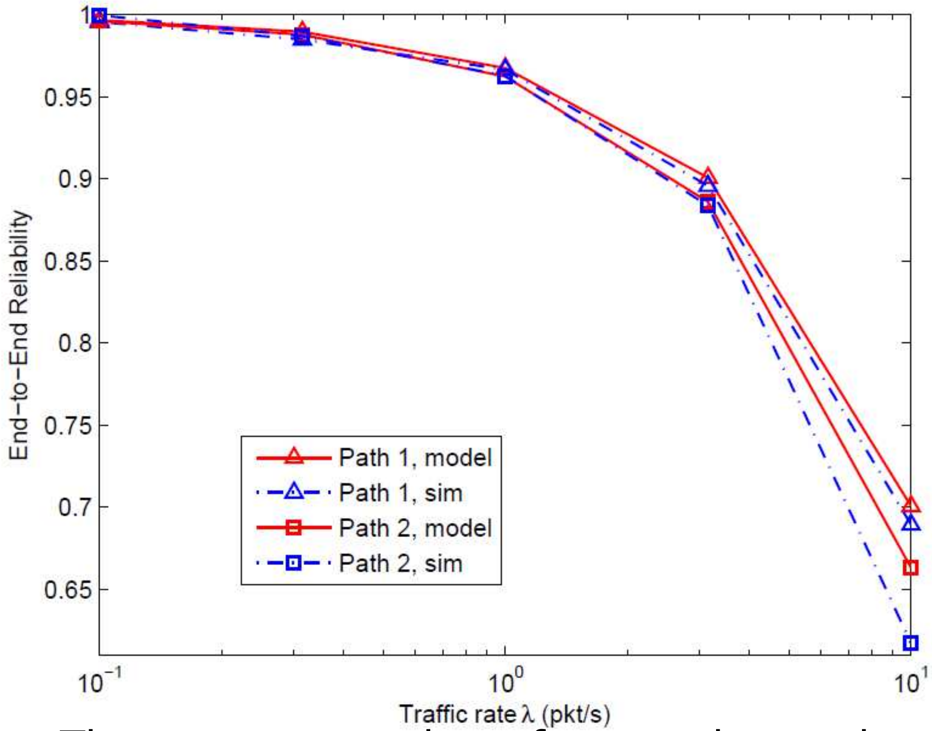
Reliability Analysis



Isolated paths



Coupled paths



Number of contenders in each time unit is lower in Path 2 → better performance

The average number of contenders reduces by forwarding packets to Path 1

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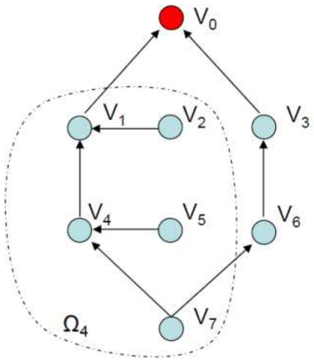
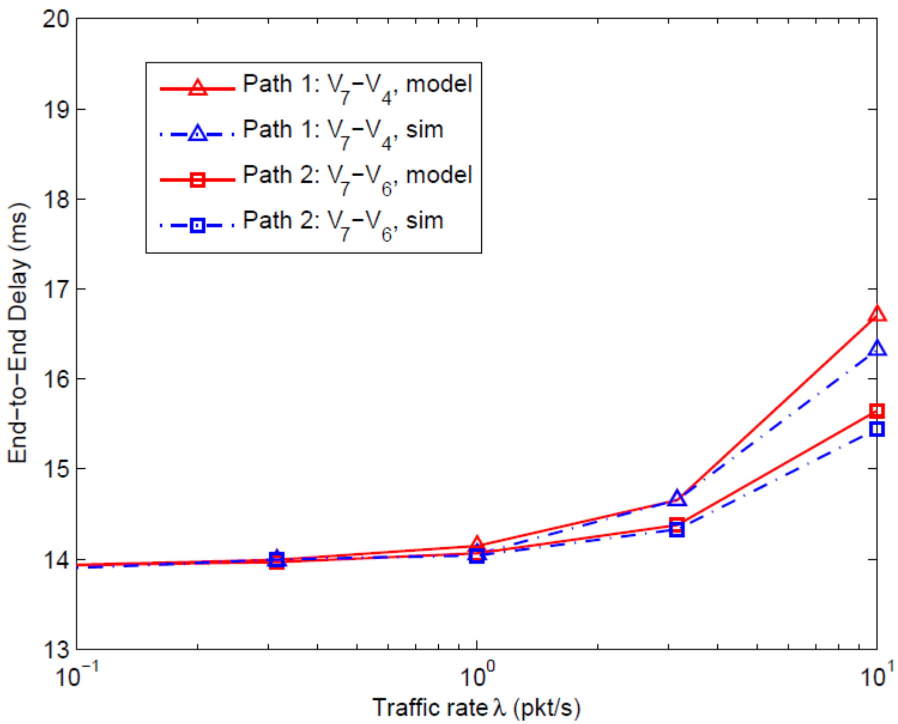
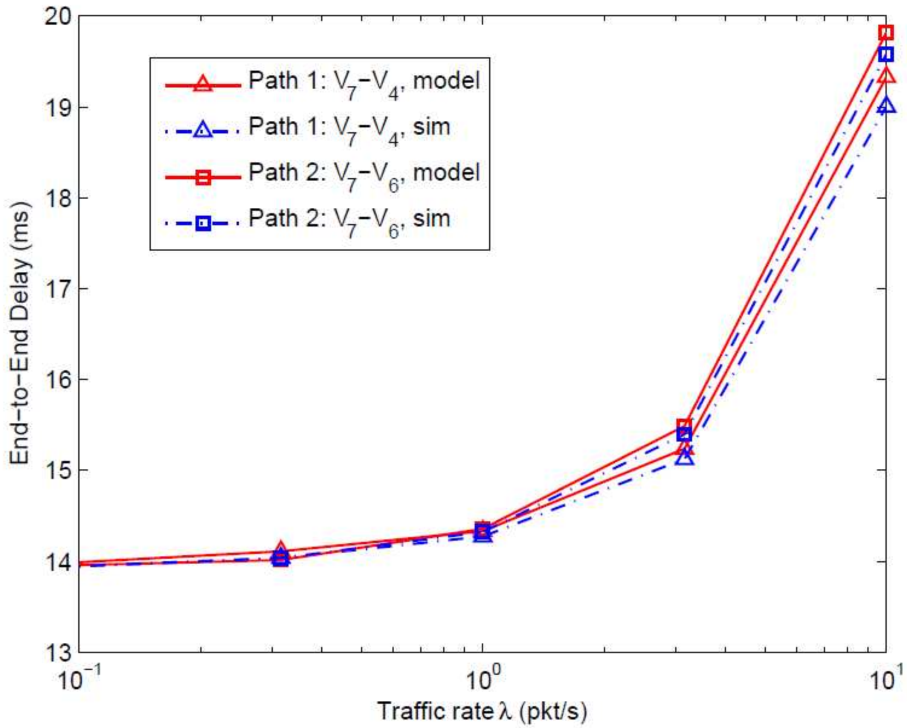
Delay Analysis



Isolated paths



Coupled paths



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Considerations

- Reliability and delay at MAC layer impact on the performance of routing algorithms
- Metrics based on reliability and delay may lead to unbalanced traffic distribution, by forcing the the forwarded traffic to dominant paths.
- Open problem:
  - How to prove the unicity of the solution of model equations

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Thanks for the attention

Questions?

Reference:

- P. Di Marco, P. Park, C. Fischione, and K. H. Johansson: *Analytical Modelling of IEEE 802.15.4 for Multi-hop Networks with Heterogeneous Traffic and Hidden Terminals.* IEEE GLOBECOM 2010.

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